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Manipulative and visual model based activities for the Algebra One curriculum

Sara Valancy
California State University, Monterey Bay

Sara Valancy
LS 400
Dr. Scott Waltz

Project Proposal

Introduction:

The problem being addressed is that many students in Algebra One lack the mathematical maturity to effectively tackle the abstract nature of the content. At a local high school, the failure rate in Algebra One is extremely high due to a lack of readiness and motivation of the students. The enormous amount of material that needs to be covered does not allow the teachers to explore alternate activities. The outlined project will meet these needs by providing short and easily implemented manipulative based activities or visual models that will introduce various algebra topics. (A manipulative is defined to be "any of various objects designed to be moved or arranged by hand as a means of developing motor skills or understanding abstractions, especially in mathematics." (dictionary.com)) These activities will help to scaffold the learning from basic skills to the abstract nature of algebra, breaking down the material, and accessing multiple forms of students learning.

Background and Motivation:

Working at a local high school in Algebra One classes, I see many students struggle with the sudden abstract nature of algebra. Taking the math 308/309 courses here at CSUMB really introduced me to the idea of manipulatives and visual models being used in math to bridge the gap between a procedure and a true understanding of the process. I feel that many of the problems students have with algebra could be eased by the incorporation of manipulatives, which often seem to stop being included in the curriculum at those higher levels of math.

Specific nature of the problem:

This problem is affecting Algebra One students across the country, as the implementation of Algebra for All is taking affect. This act encourages the push for all students to complete algebra in eighth grade. Lynn Arthur Steen of St. Olaf College discusses a few of these issues in his article from *Middle Matters,* the newsletter of the National Association of Elementary School Principals, Vol 8, No. 1, Fall 1999, pp. 1, 6-7. She states,

- Relatively few students finish seventh grade prepared to study algebra. At this age students' readiness for algebra--their maturity, motivation, and preparation--is as varied as their height, weight, and sexual maturity. Premature immersion in the abstraction of algebra is a leading source of math anxiety among adults.
- Even fewer eighth grade teachers are prepared to teach algebra. Most eighth grade teachers, having migrated upwards from an elementary license, are barely qualified to teach the mix of advanced arithmetic and pre-algebra topics found in traditional eighth grade mathematics. Practically nothing is worse for students' mathematical growth than instruction by a teacher who is uncomfortable with algebra and insecure about mathematics.
- Few algebra courses or textbooks offer sufficient immersion in the kind of concrete, authentic problems that many students require as a bridge from numbers to variables and from arithmetic to algebra. Indeed, despite revolutionary changes in technology and in the practice of mathematics, most algebra courses are still filled with mindless exercises in symbol manipulation that require extraordinary motivation to master.
- Most teachers don't believe that all students can learn algebra in eighth grade. Many studies show that teachers' beliefs about children and about mathematics significantly influence student learning. Algebra in eighth grade cannot succeed unless teachers believe that all their students can learn it. (Steen)

All of the teachers I interviewed cited the lack of mathematical maturity as a reason for the high failure rate of their Algebra One students. This is something that will be further challenged as Algebra For All comes into effect. As the veteran teacher I interviewed, who is also the head of the math department stated in my interview with her,

> <u>Myself:</u> How do you think math education has changed recently and how do you feel about these changes?
>
> <u>Teacher:</u> We no longer have the opportunity to slow things down for lower achieving kids (i.e. Algebra 1A and 1B). The middle school does not teach basic skills to the degree that they used to. Everything is based on trying to improve scores on state tests so every freshman (ready or not) is in algebra or geometry.
>
> <u>Myself:</u> What do you feel would improve students' success in math nationally? In Algebra specifically?
>
> <u>Teacher:</u> Support classes help but I really think we shouldn't try to push students into the class before they are ready. As math teachers we know that algebra 1 is the most important math class that they'll ever take. We need to make sure that they have some number sense and basic competency before placing them. (Phillips)

All three teachers shared similar sentiments, all specifically saying that the lack of basic math skills was the biggest problem facing algebra students today. Without these skills, the algebra content is nearly impossible, especially when the student is not mathematically mature enough to see beyond the abstract nature of the content.

Importance:

As an intern in multiple Algebra One classes at a local high school, I have been in each classroom at least twice a week since August. As the year progresses, the student apathy seems to be increasing exponentially. As the third quarter is halfway over and grades are being posted, the students are beginning to fight back, especially following their disappointing semester grades. Many students are already taking this class for the second or even third time and are already giving up at this point in the year. Many simply say to me, as I encourage them to work during class, 'what is the point, I am failing anyway.' This creates a society of students who feel disconnected from academic society, are unwilling or unable to finish high school, and/or have the lasting trauma of math anxiety.

Literature Review:

To address the problems that students have with Algebra, programs are being developed. Of the several state and district adopted Algebra texts and programs, one program that incorporates the use of manipulatives is College Preparatory Mathematics or CPM, (www.cpm.org). Many of the CPM Algebra activities include the use of such manipulatives as algebra tiles, which are square and rectangular tiles that are used to represent variable polynomial expressions. An overview of the program describes the development of the curriculum and the core beliefs of its founders:

> The writer-developers of CPM began with the belief that the primary goal of teaching mathematics should be ***long-term knowledge***. If learning does not persist past the end of the chapter or the end of the year, in what sense has the student learned anything useful? So the question became, what are the most effective ways to foster long term learning? Ultimately, the program was built around three fundamental principles informed by both theory and practice…The major change was to shift the focus of the student activity from being ***told a method*** or approach to being ***asked to solve problems*** designed to develop the method. The problems are attacked both individually and as a group with ideas freely exchanged as students grapple with new ideas or extensions of old ideas with the teacher as the ultimate resource. The mathematics is the same—students learn how to factor polynomials, for example—but they emerge from the CPM program with a deeper understanding of the topic and a better appreciation of where it fits into the whole structure of mathematics. (www.cpm.org)

CPM has done extensive research on the effectiveness of their program. The results of this research is described below:

Algebra 1 results

CPM schools equal or exceed the state results for students "proficient and above" in all four grades for all four years. Results in the 8th grade average 59% higher and results for the 9th grade average 31.5% higher.

2007

Grade	Test	All CA Schools	CPM Schools	% points above (below) state average	Ratio of CPM/State
8	CST Algebra I	38%	54%	16	1.42
9	CST Algebra I	17%	22%	5	1.29
10	CST Algebra I	8%	10%	2	1.25
11	CST Algebra I	5%	6%	1	1.20

SAT9 Test:

- In all five years in all three grades the CPM high schools had higher average scores, ranging between 5.8 to 10.2%, than the averages of all high schools (per county and the entire state) in California.
- Since 1998, the number of CPM high schools that score more than 10 points below the California NPR average has decreased from 25 to 11; the number that score more than ten points above the average has increased from 48 to 64. (www.cpm.org)

During my interview, the first year teacher had the follow to say about what should be done to address the problem of the high failure rate of Algebra One students,

> For Algebra specifically if the standard were to change from covering a wide breadth of knowledge to a narrow field but understood in greater depth would help algebra. There is so much to teach, that when a student learns something new they have to know it well enough to apply it the next day, but with no time to go into a topic in depth if a student misses one concept there can be a domino effect on their understand on the following topics. (Willig)

Project Description:

Partnering with a local high school, I plan to create simple, short, and easily implemented manipulative-based activities or visual models that will introduce various algebra topics. These activities will follow the lessons in the book and will not require excessive time or money to implement. The easy to implement nature makes these activities unique from other activities that require a large amount of class time, something that all of the teacher I surveyed had concerns about. These activities add to the current programs that are trying to make algebra more accessible to all students, such as CPM, because they bring the benefits of that program into all schools. With a state adopted curriculum, standards, and pacing guides, the teachers do not have the freedom to implement a full program.

Project Details:

As an academic intern at the high school, I am in an Algebra One classroom nearly every day of the week. I am able to see and work with the students and teachers and assess what activities would help them the most. As I already have permission to be in the classroom and have access to the Algebra One book, there is no extra permission needed. Since I am already with them, providing a service of extra help in the classroom, I can cater my help and activities to meet their goals. Through the manipulative based activities I have in conjunction with my mathematics capstone, I have also come to better understand that goals of the teachers I work with and how I can work within their goals. I plan to periodically check in with the instructors and have them give quick feedback on my activities. They will be asked if this is something they feel would help their students and if they feel it would be possible for them to incorporate into their lessons. I will also be working and discussing these activities with my Math Capstone advisor, Dr. Michael B. Scott.

Many of the materials I have used to perform the manipulative activities I have created or purchased on my own. A few I have been able to borrow from the CSUMB math department. In order to make these smaller, introductory activities, I will need to create an example kit, which will include examples of any manipulatives, in addition to a booklet of activities. I plan to create as many of the activities as I can, or include example manipulative worksheet cutouts that can be easily copied by the instructor. Other needed manipulatives will be purchased via on online educational store. I am hoping to receive the Alumni Association Capstone Grant to cover material and copying costs.

At the end of the semester I will be delivering to the high school a booklet of introductory activities. These activities will include manipulatives and follow the Algebra One book currently in use at the school. The end result of the project will produce a set of supplemental curriculum that can be used by the teachers in the following way. The activities will be in the form of mini-lesson plans. Here is an example:

Title:	Quotient Rule of Exponents
Materials:	Any set of small objects (beans, plastic disks, paper x's, etc)
Procedure:	• Review exponents and what they mean • Hand out the materials to each student

	• Write an introductory problem on the board (ex. x^3/x^5) and have the students place the objects on the desk to model the problem (using a pencil or pen as the fraction bar) • Ask the students to simplify the problem and try to figure out what the rule is • After time has passed, ask students to demonstrate what they have found • Show the students how to reduce by making small fractions of object over object (1/1) • Continue with more examples
Intention:	The intention of this activity is to try to scaffold the students from the idea of division and that anything over itself is one. This activity helps them to discover and see the rule in action, before it is formally introduced. This is also a good way to asses whether your students understand the idea of an exponent as they model the problem, (ex. understanding that x^3 should look like x x x). Since many students often leave one x either on top or bottom while reducing, if possible it would be good to include a #1 for each student so that they could leave something on each side of the fraction.

Timeline:

2/22-2/28	Begin retrospective paper, begin looking through Algebra One book to pick sections that can be made into activities
3/1-3/7	Finish retrospective paper, make instructor appointment, continue gathering activities
3/8-3/14	Begin writing up activities, begin final paper outline
3/15-3/21	Spring Break
3/22-3/28	Finalize outline, begin writing up paper, continue to write up activities
3/29-4/4	Meet with instructor, continue to work on paper, continue to write up activities
4/3-4/11	continue to work on paper, continue to write up activities
4/12-4/18	Finish writing activities, check in with community partner, continue to work on paper

4/19-4/25	Make activities into a booklet and gather any materials that will need to go with it, draft final reflection paper
4/26-5/2	Revise final reflection paper, begin slideshow for Capstone Festival
5/3-5/9	Finish project and hand in, archive capstone, complete slideshow
5/10-5/16	Present at Capstone!

To success will be measured through the feedback I receive from the teachers at the high school. I have also had experience in what will be a successful activity from the manipulative-based activities I have been performing in one Algebra One class as part of my math capstone. I have been able to study student reaction and behavior during this activity, as well as the teacher's reaction and willingness to participate in the activity. I hope to also use some of these activities in my private tutoring as I write them and test to measure their success.

Conclusion:

In conclusion, the failure rate of Algebra One students at a local high school is truly astonishing and the teachers are scrapped for time and overwhelmed. Manipulatives are a great way to reach all students though enacting all learning styles and to address the needs of English Language Learners. Manipulatives also help students to understand the abstract nature of algebra by scaffolding through lower level math, while also reviewing those concepts. Creating these activities will give the teachers an easy way to bring manipulative into their algebra one classroom.

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Appendices:

- Survey with Veteran teacher and head of the math department at the high school:

Survey Questions:

What do you feel are currently the biggest challenges for Algebra One students across the nation? Lack of basic skills (add, subtract, multiply, divide), maturity needed to face multi-step problems, inability to think abstractly.

How do you feel these issues affect students at Seaside High? We have an extremely high failure rate in algebra 1. Many lack the skills to adequately deal with the material. Many lack the work ethic it takes to succeed.

How do you think math education has changed recently and how do you feel about these changes? We no longer have the opportunity to slow things down for lower achieving kids (i.e. Algebra 1A and 1B). The middle school does not teach basic skills to the degree that they used to. Everything is based on trying to improve scores on state tests so every freshman (ready or not) is in algebra or geometry.

What, if any, issues do you feel are particular to Seaside High Algebra One students?

It is hard to say what problems are unique to SHS.

What do you feel would improve students' success in math nationally? In Algebra specifically?

Support classes help but I really think we shouldn't try to push students into the class before they are ready. As math teachers we know that algebra 1 is the most important math class that they'll ever take. We need to make sure that they have some number sense and basic competency before placing them.

Do you think any of these suggestions would be possible to implement at Seaside High?

Do you feel hands-on, manipulative based activities would be beneficial to incorporate into the Algebra One curriculum at Seaside High?

To some extent, but there really isn't a lot of time for that. We have to keep moving to get all of the standards in. Also I have found that students have trouble applying the concepts that they learned from manipulatives.

- Survey with first year teacher

Survey Questions:

1. What do you feel are currently the biggest challenges for Algebra One students across the nation?

Inconsistent basic skills (operations, fractions, order of operations, etc.) create a barrier to the higher level skills needed to be successful in algebra. Also this low skill can create a dislike or fear for the math. This often stems from students being put through a class that they are not prepared for, so they are not scaffold into classes where success is a foreseeable outcome.

2. How do you feel these issues affect students at Seaside High?

This dislike for math and the lack of basic skills create form the habits of not attempting work because they already feel that they cannot succeed.

3. How do you think math education has changed recently and how do you feel about these changes?

The focus, not only in math, is to prepare students for college which means Algebra 1 is the only choice for students. The question is can they get into college, as opposed to: what is the best direction for the students' education. The desire is to send a student into college is <u>an</u> answer to the question of the best direction; however that isn't the only answer we should have. Should we force one path for students even though having a financially applied math class might create a better prepared citizen and create an atmosphere of where education can be a useful thing? Those who are forced towards college but have no desire or drive to reach college might perform better in their education if they can relate to the goals that are set.

4. What, if any, issues do you feel are particular to Seaside High Algebra One students?

Algebra 1 students at SHS are faced with large classes, which reduce the amount of 1on 1 attention they can receive. There are also very few resources for the algebra 1 classes to have differentiated instruction with technology and kinesthetic materials.

5. What do you feel would improve students' success in math nationally? In Algebra specifically?

The greatest factor for students' success in mathematics would be to have standard based classes that are not utterly dependent algebra skills. This would increase student success & match the standards to more possible outcomes for the students, which would create a greater general understanding and appreciation for math.

For Algebra specifically if the standard were to change from covering a wide breadth of knowledge to a narrow field but understood in greater depth would help algebra. There is so much to teach, that when a student learns something new they have to know it well enough to apply it the next day, but with no time to go into a topic in depth if a student misses one concept there can be a domino effect on their understand on the following topics.

6. Do you think any of these suggestions would be possible to implement at Seaside High?

Currently, SHS has an informal geometry class that has no identified goals and standards, the teachers of Informal geometry could work together to create the goals and standards for the class and focus the instruction on understanding the topics in depth instead of a breadth of topics.

7. Do you feel hands-on, manipulative based activities would be beneficial to incorporate into the Algebra One curriculum at Seaside High?

Yes, with manipulative based activities the class instruction could be differentiated more effectively and allow teachers to access different modes of student learning.

8. What, if anything, have you learned about the incorporation of manipulatives into mathematics in your education or training?

Manipulative based activities in the class can differentiate instruction and allow me to access different modes of student learning. I have also learned using manipulatives can be very difficult in class and often the written or measured outcome of the lesson is not provided or easy to construct. That is: if I use algebra tiles in class and want students to record what they did, I have to make the worksheet myself and it usually takes me several tries to find one that works well.

9. What are your concerns or hopes about incorporating manipulatives into Algebra One?

I hope that I will someday have the resources to have manipulatives available to me, but more importantly I hope that the pacing guide has days built into the instruction to allow time to explore and use these manipulatives effectively.

- Survey with teacher with intermediate experience, who is also a police officer

Survey Questions:

1. What do you feel are currently the biggest challenges for Algebra One students across the nation?

   I feel the first problem is basic numeracy: simplifying, fraction reduction etc, etc. Beyond this, basic problem solving – mathematical or non-mathematical - to improve cognizance would help students in Algebra greatly

2. How do you feel these issues affect students at Seaside High?

Students are not encouraged by society to problem solve. Being connected to all types of electronics at all times doesn't help students to concentrate

3. How do you think math education has changed recently and how do you feel about these changes?

All students are not ready for Algebra I by their freshman year, though all have to take it. I believe we should offer more levels to accommodate students' abilities, therefore, help them progress

4. What, if any, issues do you feel are particular to Seaside High Algebra One students?

Same as above. They are not ready for the patience in problem solving required for success in Algebra I.

5. What do you feel would improve students' success in math nationally? In Algebra specifically?

   Do not make it so easy for them to escape mentally from problem solving. Also, break Algebra I into different levels that are appropriate for students' needs.

6. Do you think any of these suggestions would be possible to implement at Seaside High?

   Not with the current budget crisis or with the current administration in the school district or at the state level.

7. Do you feel hands-on, manipulative based activities would be beneficial to incorporate into the Algebra One curriculum at Seaside High?

   I do, though there is little time for this when the curriculum states that we have to get through twelve chapters and take mandated state tests. Basically, no time.

8. What, if anything, have you learned about the incorporation of manipulatives into mathematics in your education or training?

   They can help students see the usefulness of concepts as they apply to the real world, such as the travel project for graphing that I have done in the past.

9. What are you concerns or hopes about incorporating manipulatives into Algebra One?

   That we have enough time to use them and that they are used so students may increase their interest in math and problem solving capability.

<u>Final Reflection Paper</u>

As Frank Herbert once stated, "The beginning of knowledge is the discovery of something we do not understand." (www.quotationspage.com) After taking the Math from and Elementary Viewpoint class series at CSUMB I could not understand why such a fantastic tool like manipulatives and visual models were not more utilized in upper level math classes. Upon entering multiple Algebra One classes at a local high school for my job as an academic intern, I was really surprised at the huge amount of lecture that took place in the class. There was virtually nothing done to address the learning styles of the students and there was little interaction and self-discovery being done between the students and the material. I used this capstone project as an opportunity to discover what manipulatives were currently in use for algebra and how I could most easily and effectively implement them.

The first part of my project involved taking four periods of algebra one at a local high school and implementing manipulative based activities with two of the periods approximately once a week. This project was significantly more difficult than I had anticipated. There were many unanticipated factors out of my control that affected my ability to perform the activities as I had expected. Primarily, I was not in the classroom on a daily basis, leaving only two days out of the week left for me to create and perform lessons. Many times these days were taken up by other commitments, such as quizzes, tests, or teacher training. Of the options I had, it was very difficult to create an activity using manipulatives that would not only be effective, but also physically possible and cost effective in the given amount of time. As the schedule was in constant flux, the time I had to create and prepare each activity was often very limited.

With limited options for lessons to choose from, as I was only in the class for extended time two days a week, incorporation of manipulatives into these lessons proved to be very difficult. I often had an idea that would incorporate manipulatives, but was not possible to set up and perform in the time allotted, or was not financially possible. Often, as in the case with algebra, the concepts are very abstract and hard to translate to concrete form. I had difficulty thinking of an idea to change a topic that was presented in the book is such a structured and rule-based way without any examples or guidelines to follow.

During the time that I was in the classroom, the instructor and I had very little opportunity for communication on what each activity would include. Often he would give me a lesson, and I would sketch out a description of my idea to run it by him. Many times, the lesson that I was told to follow was too advanced for the level the students were at. For example, in one particular case the teacher told me to take example problems from the book for a particular lesson. When I came, expecting the students to have been prepared for the activity the previous day, the students could not do a single problem. They were struggling so severely with the beginning steps necessary to set up the problem that they were prevented from completing the lesson. In the end, my activity had to be completely thrown out and the implementation of the manipulative was entirely overshadowed by the difficulty of performing basic operations. Having a lack of control and awareness of the students' preparation level, I walked into the class with a set of expectations given from the book that was completely unrealistic. Although this was a difficult lesson to learn, it was a necessary one that taught me a lot about curriculum development.

Although I have had a lot of service learning and tutoring experience as an undergraduate, I have never prepared and performed lessons for large group of students at this high of an age/academic level. The experience of teaching alone was a daunting task, just getting used to being in front of 30 sets of eyes! What made the experience even more difficult was that I was not the teacher, so the students made comments that made me feel as though they saw me as more of an equal and less of someone that is too be respected. On top of having a new teaching style, I was also asking the students to do things they had never been asked to do before. These requests included working in groups, using manipulatives, and sharing their results with the class. As I was not the instructor, I had no power to enact any discipline or classroom management. The days of the activity, as well as the days before in preparation, were not in my control and the activity was often affected by my higher expectations of the class' level of preparation.

As this new type of learning was introduced, there were many ups and downs. The very first activity was the most difficult, as it asked the most of the students as far as participation. I asked for the students to get out of their chairs, so I could move the desks to the side. I placed the students in teams, and asked them to participate one at a time per team. Almost all of the students were unwilling to participate and get out of their chairs. The teams disassembled almost immediately and some students even refused to get up at all. Their lack of motivation made the benefits of the activity impossible to reach. This was often true with the activities where my level of expectations did not match up to the students' preparation level as well. Frustration set in fast and then the activity became pointless.

The manipulatives were sometimes a distraction to the students, but giving the students a minute to get used to the activity seemed to get most of the distraction out. Though the students were sometimes playing with the manipulatives, they were at least engaged in the activity. The teacher commented on how even if the students were playing with the manipulatives, then at least they were doing something with their hands at that activity may be what they need to help them stay focused and connected to the activity.

With manipulatives, another problem that arose was how to test the outcomes of the lesson. The students often seemed to feel as though the activity was optional when nothing was being collected or documented. With the help of the teacher, the solution often seemed to be some sort of accompanying worksheet that the students could fill out as they did the activity. This was a way to document and draw what they had explored with the manipulatives.

The entire year in this class has been a struggle between class and teacher, grades being the biggest indicator. The grades before were very varied, as many of the students have taken algebra one before and most were very familiar with beginning lessons. For the first semester, the teacher did not require the class to take notes and the only graded activity was homework. Many times the students would sleep through the 30-45 min lecture and then lag in starting the homework for the remaining time of the hour-long class period.

The students seemed to feel very little responsibility for their grades and often had no idea as to what their grade was at any given time. As this was the case, the semester grade was a shock to many of the students. Large majorities, average of about 20 out of 30 students per period failed that class at the semester and still are in this last week of the third quarter. The different skill levels of the students, different experience levels and large numbers of students who are absent for long periods of time make it nearly impossible to track grades as an indicator of progress. Additionally, the fact that I was not in classroom on a daily bases doing an activity made it so my interaction would have very little effect on overall class grades.

Though there was little affect on the students overall class performance, their behavior during the activities and their comments on an ending survey showed there was some measurable success. The most telling question on the survey was as follows:

> Did you feel the activities that used the hands-on tools (like the geoboards with rubber bands and algebra tiles) helped you better understand the lesson? Did you think about them while taking the test or doing homework? Please describe why and how.

In period one, 11 students found the activities helpful, where as 9 did not. In period three, 17 students found them helpful, where as only 5 did not. The difference in periods 1 and 3 can easily be explained by the fact that period one was the test period, where the activity was performed for the first time. Following the reaction of period one, the teacher and I would adjust the activity for the following period. If the activity simply was not successful in implementation, we would not do the activity for period 3 at all. Therefore, period 3 always experienced activities in a smoother and more successful way than period one did.

The comments left on the after survey questions were the most telling and helpful part of the entire survey process. Of the students who left comments on the above question, 6 were positive and 5 were negative. In period 3, 9 were positive comments and one was negative. The following chart reports the comments for period one:

Positive Comments	Negative Comments
• Makes the material more fun • Makes learning more interesting • Helped me in my homework • They let me see things more clearly • It helps a lot better because if you need help you can ask the person sitting next to you because the whole class is doing the same thing and solving the same problem at the same time. • We should do more activities so we can learn and it can make it easier for us to learn.	• Instead of helping me it made me forget how to do it • Very boring • Like being in kindergarten • Confusing

The following chart reports the comments for period three:

Positive Comments	Negative Comments
• I think that was the easiest because it is didn't get it someone would help me. • It showed and you got to plan and I understand that better than just writing it. • They helped me because we learned how to actually do them and not just being lectured about it. • It is easy for me to learn it faster and I can see my mistakes. • It really helped me understand better. • I can learn better by touching and moving because it gives you a better way to try it and learn it faster • We had fun and we learned better • It helped me understand the material better • Its easier because its better to help understand it and its not as boring as the others (auditory and visual) • The geoboards really helped me solve and graph equation	• I didn't understand it

Along with these comments, many students also showed behavior that implied the activities were making a difference. The students expressed moments of understanding from the students and myself. Many of the students would make comments that they finally understood or that the material was easy. During each activity, there would be a minimum of one moment where I would see the manipulatives working for at least one student. As a whole, the class would show significantly more overall participation during the activity periods than during the lecture periods.

As time went on and the more activities I did, the more experience the class had. They therefore appeared more prepared for the types of activities I would ask of them and my teaching style. By the final activity, the level of complaining had decreased significantly and nearly all of the students were participating in the activity all or most of the time. From the comments they made to me, they seemed to finally understand the purpose of the manipulative is not a long-term method, but as a visual explanation to produce deeper understanding.

The overall experience of this project has been very fulfilling and informative. Personally, it has been a very challenging undertaking. It was difficult to approach teaching while simultaneously trying to bring in a new technique. In the end, I came away with a set of results that I feel are very useful for the future and helped me to decide on how I would create my introductory activities to leave at the school site.

The first and foremost, I learned that adding manipulatives and visual models to algebra can be done and, if done correctly, does have a positive impact. The inclusion of manipulatives definitely helped the students to see why each formula worked and to get a visual interpretation of a usually very formula-based subject. I observed that the students were often more engaged in the activity when using manipulatives than with purely a lecture format. The teacher seemed to come alive when instructing with manipulatives, being forced to explain the reasoning and background behind each topic to develop a formula and a process.

However, I learned most of all the caution must be taken when using manipulatives. The manipulative can easily become a distraction to the students if not introduced in a very structured form. Due to the fact this is something that the students are not used to, they can feel overwhelmed as if it is something else that they have to learn and are intimidated by developing a new skill. I found that if the intention of the manipulative is not properly introduced as a learning tool, and not as a new process for performing the operation, the students see it as something extra they can ignore.

All of these factors went into my decision to make the activities as I did. First, I experienced the large amount of work and preparation that it took to do full 30 minute or longer activities. They became a huge undertaking for me as the teacher and were difficult to implement into the already packed curriculum. The teachers are stretched thin as it is, and the survey I gave at the beginning of the semester implied that they were leery of adding yet another aspect to their day. This is why I choose to do introductory activities, where the manipulatives and visual model could be used to introduce a new concept or topic. The manipulatives must be seen first, so the students can work from the concrete to the abstract. They provide not only a visual explanation of what is happening, but also a reason to utilize the more accessible formula.

When planning the activities, I also was careful to create lessons where the students could do the work individual or in pairs at their desks. Without a prior expectation for group work and sharing with the class, the students seem unwilling to share and make themselves vulnerable to the class. For my future as a teacher, I have learned that group work, sharing, and the expectation of participation is something that must be a constant, daily part of the classroom from day one. This way it is expected, and does not catch the students' off-guard.

Another important lesson I learned is that the manipulatives must be a constant theme in the class, from start to finish. I saw that it would work best to have had the same manipulatives utilized many times in different ways throughout the year. This way the novelty of the manipulative wears off quickly and the students are already aware of how to use it without a lot of instruction. Luckily, I was recently introduced to a set of algebra model manipulatives in one of my classes at CSUMB. They show not only why unlike terms cannot be combined, but also model all other operations with polynomials that regular algebra tiles also do. This has allowed me to create lessons that use the same manipulative repeatedly throughout the year.

If I were to create the activities again, I would have found the algebra models earlier so that I could have tried to implement them in the classroom throughout the entire year. I would have also liked to have more of a role in the school, so that I could have worked more closely with the teachers and received more feedback on the success of the activities in their classrooms. What will measure the success of the activities in the future is the amount of use they receive. I tried to be most conscious of making the activities accessible and easy for the teachers to use. Hopefully I was successful in this area.

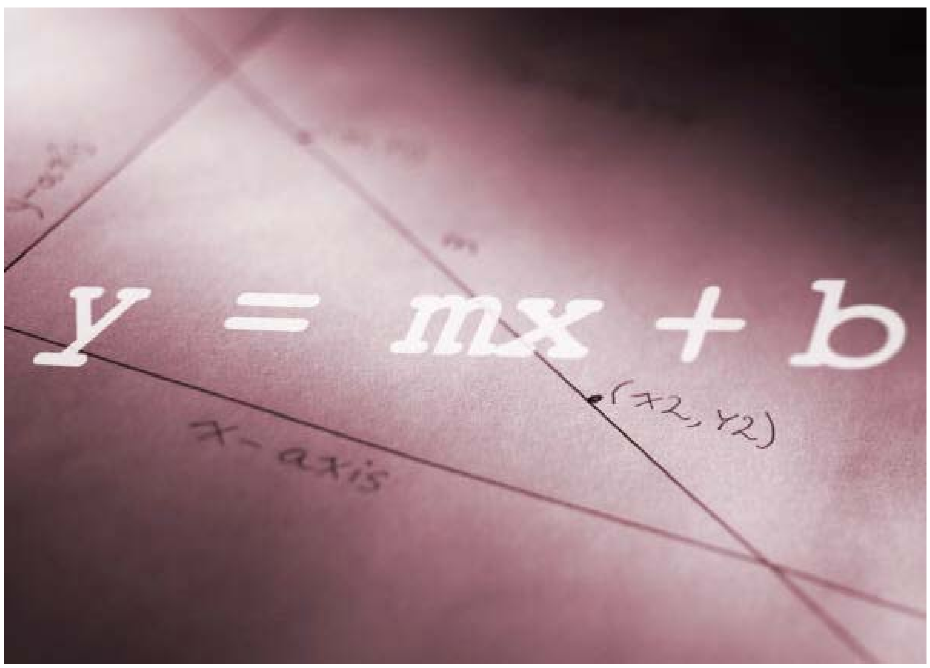
What I do know is the feedback I have gotten so far from the teacher I worked with. He expressed via interview his intention to continue using manipulatives and enjoyed how I brought them into his classroom. With these shorter, easier activities and the materials to accompany them, this will be possible for him and the other teachers to carry out. I would tell all the teachers, or anyone else attempting to implement manipulatives, to be most conscious of making the activities easy to use for both students and teachers.

In the future, I plan to further explore manipulative implementation for my master thesis. I want to try this again in a fuller and more intense form, using all that I have gained from this project. The overall experience was gave me an enormous amount of insight into student motivation, the use of manipulatives, and mathematics education in general. The most important lesson is that manipulative implementation is possible and successful at the higher-grade levels. Now I finally know that I can bring the visual and kinesthetic beauty of math to the subject of algebra that I want to teach. Also, there is no reason to re-invent the wheel! There are great manipulatives out there and with educated implementation, and additional creativity to fill in the gaps, every lesson can involve something for all learning styles. In conclusion: math can be fun!

Works Cited:

Herbert, Frank. "The Quotations Page." *The Quotations Page*. 28 May 2009 <http://www.quotationspage.com/quote/26173.html>.

Manipulative and Visual Model Based Activities for the Algebra One Curriculum



By,
Sara Valancy
Spring 2009
Dr. Scott Waltz

Introduction and Purpose

The purpose and intention of these activities is to provide you with what I hope is an easy way to experience the benefits of incorporating manipulatives into your classroom. These activities were created as a result of a final undergraduate study on trying to incorporate manipulatives into the Algebra One curriculum. My overall experience and the feedback I received from the students have shaped these activities into having plenty of interaction between students, teachers, and material. The whole class is working on the same problem, so the students can be encouraged to learn from each other. The manipulatives are also a great way to address the multiple learning styles of your students.

You will see that all of the manipulatives are repeated throughout the lessons, to create familiarity for the students. I have found that the novelty and unfamiliar feeling of the manipulative wears off the more often the students see it, and distraction is also severely deceased. Thus, these activities are intended to be used throughout the year.

Most importantly, I have learned that one should explain to the students prior to a activity that this is purely a visual representation of what goes on when performing an operation or using a formula. The manipulative help to show that there is concrete meaning to the material. This is not a method for the long run, but it is something to reference as the algorithm is being applied.

Many of these activities have been are based on those found in the Algebra Models workbook by Elyce B. Duerr, the Algebra One text by Larson, Bosewell, Kanold and Stiff, or created by myself. Special thanks as well to the CSU Monterey Bay Alumni Association for providing the resources to purchase and create the accompanying manipulatives.

Thank you and I hope you enjoy putting these activities into action!

For any questions, comments, suggestions, or improvements, please email Sara Valancy at svalancy@yahoo.com.

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Chapter 8

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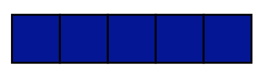
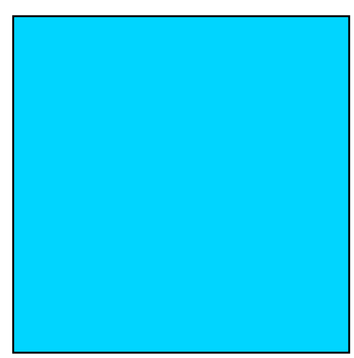
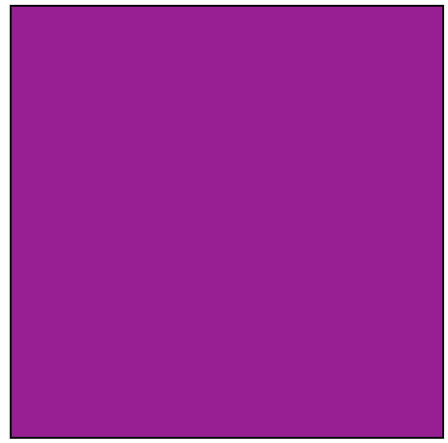
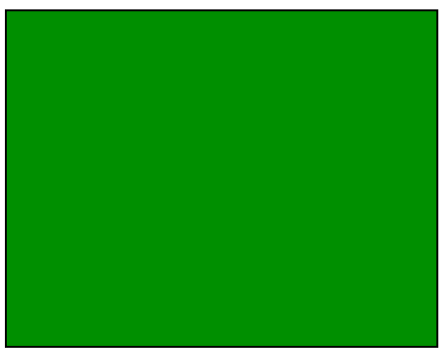
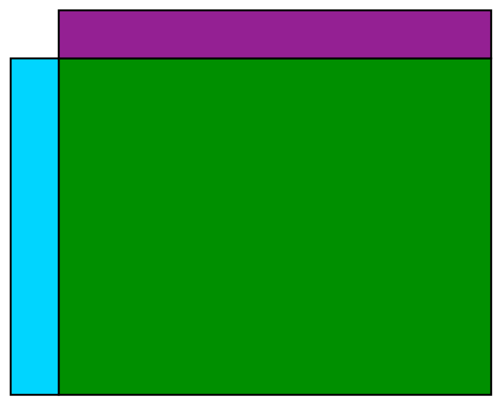
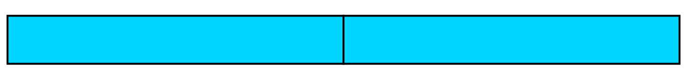
Chapter 12

Lesson 1.1

Title:	Variables in Algebra
Standards Addressed:	15.0
Length of Activity:	15 – 20 min
Materials Used:	Algebra models, overhead models (if desired)

Description:

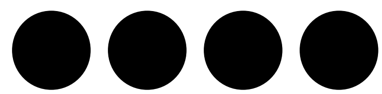
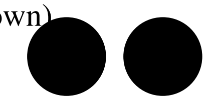
<table>
<tr><td>Introduction:
Define variable and value</td><td>Variable: “When a letter is used to represent a range of numbers, it is called a variable.” pg. 3
Value: the number that the variable represents is its value</td></tr>
<tr><td>Intro algebra models:</td><td>These algebra models are a way to physically represent variables.</td></tr>
<tr><td>Pass out algebra models and let the students explore</td><td>Have them note:
Different colors, shapes and sizes
The red side of all tiles
The common sizes of the sides of different tiles</td></tr>
<tr><td colspan="2">Introduce each tile and show how they relate:
Emphasize the fact that the single units cannot be lined up with the shapes of side x units, since that would be assigning a value to x, where x is unknown.</td></tr>
<tr><td>1 unit (1 by 1 square) </td><td>5 units: </td></tr>
<tr><td>x (1 by x rectangle)
</td><td>x^2 (x by x square) </td></tr>
</table>

<table>
<tr><td>y (1 by y rectangle)
</td><td>y^2 (y by y square)
</td></tr>
<tr><td>xy (x by y rectangle)
</td><td></td></tr>
<tr><td>Give students various variable expressions to lay out on their desks:</td><td>Examples:
2x 4 + x 3y 7 – y x – 2</td></tr>
<tr><td colspan="2">Lead into a discussion and examples of substituting values into variable expressions. Use the models as a visual representation of what is going on, then slowly move on to higher values.
For example: 2x when x=3  3 + 3 = 6
3 3 2(3) = 6</td></tr>
</table>

Lesson 1.2

Title:	Exponents and Powers
Standards Addressed:	2.0
Length of Activity:	10 min
Materials Used:	black/red counters, overhead counters (if desired)

Description:

Pass out the counters to each student or student pair	
Define key terms of a **power** such as 2^3:	**Exponent**: the 3 **Base**: the 2
This is intended to just introduce visually what a power is. Have the students explore with the counters what a power like 2^3 could represent. How could it be shown with the counters?	
After time has passed, show the students examples of powers represented with the counters. (note: color doesn't matter)	Ex: 3^4  = 4, 3's or 3*3*3*3
Have students do examples, writing the answer in number form, word form and representing with the counters, (counters can continue to be used for students who need them).	practice problems: 4^2 (shown)  5^3 x^6 3 squared x to the fourth power s cubed

Lesson 1.4

Title:	Equations and Inequalities
Standards Addressed:	5.0
Length of Activity:	10 min, longer (if desired)
Materials Used:	Felt pockets with index cards

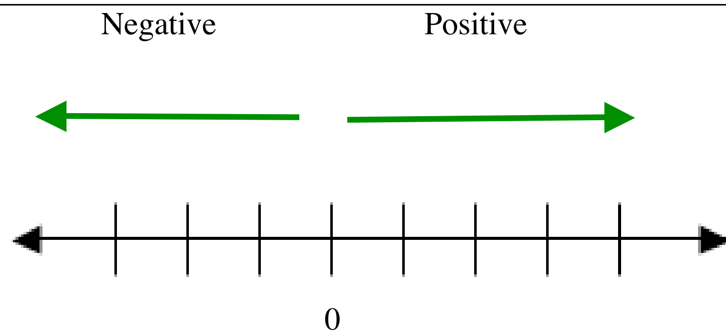
Description:

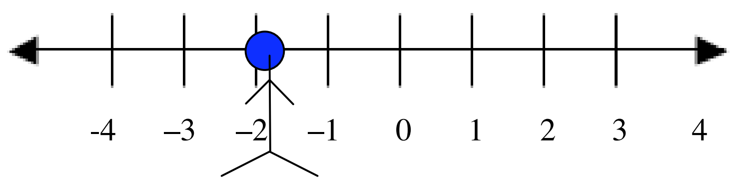
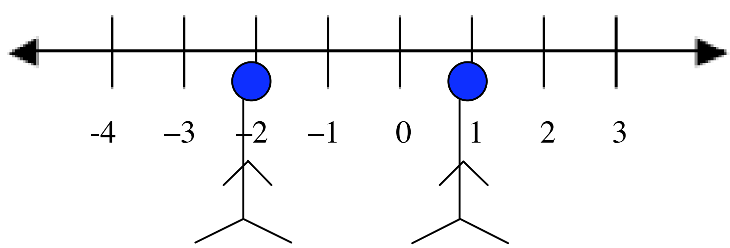
Tape or clip the felt pockets to the board. Write the equation on the board as is, with correct variable card in the pocket. The pockets are to emphasize that a variable represents a value, and what substitution means.	$3\ \boxed{x}\ + 6 = 12$
Question: Is 4 a solution of $3x + 6 = 12$? Replace the x card with a 4 card and check. $3(4) + 6 = 12$? $12 + 6 = 12$? $18 \neq 12$ not a solution	$3\ \boxed{4}\ + 6 = 12$
Question: Is 2 a solution of $3x + 6 = 12$. Replace the x card with a 2 card and check. $3(2) + 6 = 12$? $6 + 6 = 12$? $12 = 12$ true, 2 is a solution	$3\ \boxed{2}\ + 6 = 12$
Repeat the process with inequalities,	Scaffold into using mental math.

Lesson 2.1

Title:	The Real Number Line
Standards Addressed:	1.0
Length of Activity:	10 min
Materials Used:	Large walking number line

Description:

<table>
<tr><td colspan="2">Mount the number line by clipping it or taping to the top of your board, or laying it on the floor where the class can see it.</td></tr>
<tr><td>Put a marker or write the number zero around the center of the number line. Introduce the positive and negative directions of the number line.</td><td>
</td></tr>
<tr><td>• Have a student come up and stand at a specific point.
• Ex. Stand at – 2

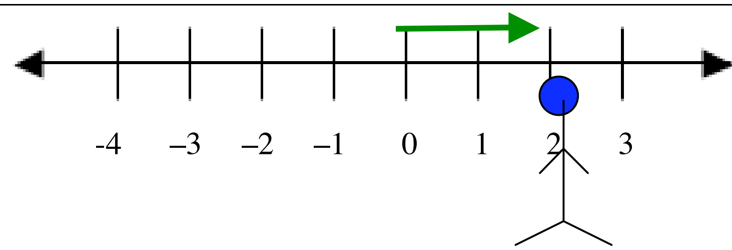
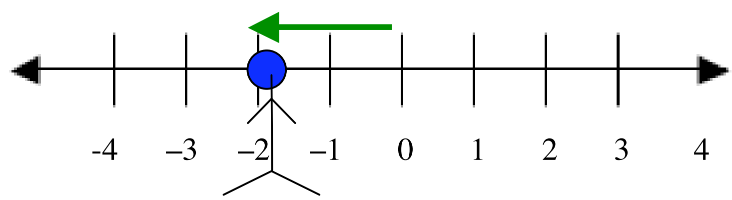
• Repeat with several students.</td><td>
</td></tr>
<tr><td>• Next, ask two students to come up, each standing at a different point.
• Ex. students A stand at –2, student B stand at 1
• After reviewing the inequality symbols, have the class help write two different inequalities for this situation.</td><td>

Have the students link arms to show a solid line between –2 and 1</td></tr>
<tr><td colspan="2">Scaffold into graphing fractions and decimals.</td></tr>
</table>

Lesson 2.2

Title:	Absolute Value
Standards Addressed:	3.0, 24.3
Length of Activity:	5 min
Materials Used:	Large walking number line

Description:

<table>
<tr><td colspan="2">Mount the number line by clipping it or taping to the top of your board. Just write the numbers on the line. Review the positive and negative directions and <, > signs.</td></tr>
<tr><td>• Have a student come up and stand at zero
• Tell this student to walk to + 2</td><td>
</td></tr>
<tr><td>• Have a student come up and stand at zero
• Tell this student to walk to - 2</td><td>
</td></tr>
<tr><td>• Ask the first student how many spots he/she moved.
• Ask the second student how many spots he/she moved.</td><td>Discuss how this is a representation of absolute value, specifically $\lvert 2 \rvert$. Absolute value is the distance traveled away from zero, regardless of the direction. Distance is always positive.</td></tr>
</table>

Lesson 2.3

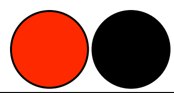
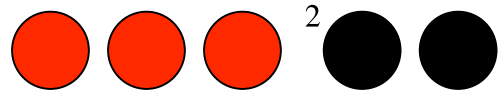
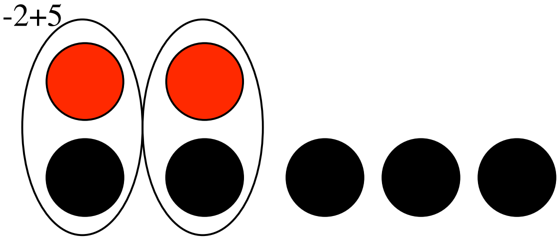
Title:	Adding Real Numbers
Standards Addressed:	1.0
Length of Activity:	10-15min each part, can do 1 or all parts

There are multiple options for this lesson, one or all can be done as you wish.

Option 1: Adding with Counters

Materials: black/red counters, overhead counters (if desired)

Description:

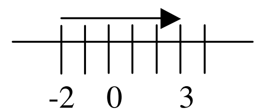
Pass out counters to each student or student pair Describe red represent –1 and black represent +1	 -1 +1
Explain the concept of a "zero" -1+1 = 0, so any pair of red and black cancel out	 = 0
Have students represent various real numbers	ex. -3 2 
Next have students represent various expressions	-2+5  0 0 +1 +1 +1 = 3
Scaffold into the idea of working with larger numbers, "Do we have more positives or negatives? So our answer will be ____, now subtract as usual, smaller from larger."	

Option 2: Using the Number Line

Materials: white boards, numbers line inserts

Description:

Pass out the page protector white boards with number line inserts, to each student. Allow them to explore the number line, discuss the positive and negative directions.

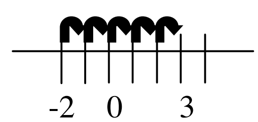
<table>
<tr><td>Walk students through a few example problems, having them physically walk their fingers from one number to another.</td><td>Ex. –2 + 5
Start on the negative 2, and walk 5 places in the positive direction. Land on the 3.
</td></tr>
<tr><td>Give the students more examples.</td><td>-4+5 -1+(-2) 4+(-5) 0+(-4)</td></tr>
<tr><td colspan="2">Scaffold into the idea of working with larger numbers, “Do we have more positives or negatives? So our answer will be ____, now subtract as usual, smaller from larger.”</td></tr>
</table>

~ Options: you can also pass out markers and have the students draw the arrows, or place + and – signs on the appropriate sides

Option 3: Walking the Number Line

Materials: walking number line (large green felt number line)

Description:

<table>
<tr><td colspan="2">Mount the number line by clipping it to the top of your board, taping it to the board, or laying it on the floor in an area where the students can see it. Just write the numbers on the line, or, if it is on the floor, lay a marker at zero. Discuss the positive and negative directions</td></tr>
<tr><td>Walk students through a few example problems, having them physically walk from one number to another. If the number line is on the board, you can show the jumping of the numbers, (shown at right).</td><td>Ex. –2 + 5
Start on the negative 2, and walk 5 places in the positive direction. Land on the 3.
</td></tr>
<tr><td>Have the students perform more examples.</td><td>-4+5 -1+(-2) 4+(-5) 0+(-4)</td></tr>
<tr><td colspan="2">Scaffold into the idea of working with larger numbers, “Do we have more positives or negatives? So our answer will be ____, now subtract as usual, smaller from larger.”</td></tr>
</table>

Lesson 2.4

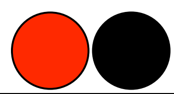
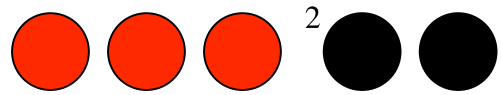
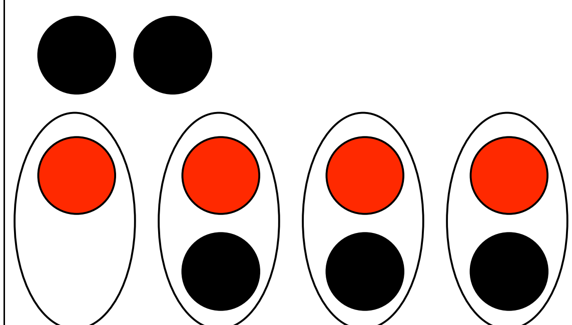
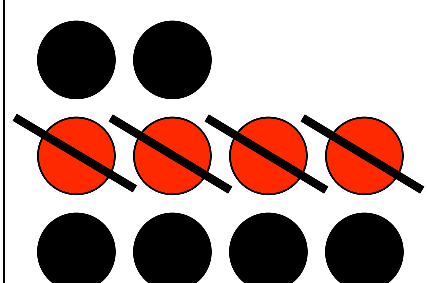
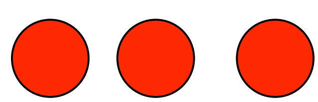
Title:	Subtracting Real Numbers
Standards Addressed:	1.0
Length of Activity:	10-15min each part, can do 1 or all parts

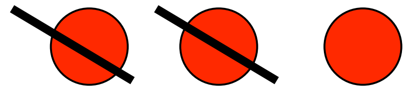
There are multiple options for this lesson, one or all can be done as you wish.

Option 1: Adding with Counters

Materials: black/red counters, overhead counters (if desired)

Description:

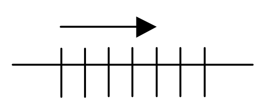
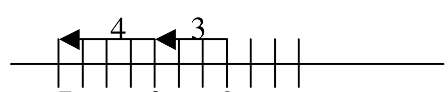
Pass out counters to each student or student pair Describe red represent –1 and black represent +1	 -1 +1
Explain the concept of a "zero" -1+1 = 0, so any pair of red and black cancel out	 = 0
Have students represent various real numbers	ex. -3 2 
There are two different forms of subtraction problems. Example of the first is 2 – 4 * Place 2 positive counters * Since there are no negatives to subtract, the only way to get zeros into the equation without changing it is to add in zero pairs. Add in four zero pairs.	
Now subtract the –4, so you are left with +6	
The second form is much simpler. Ex. –3 – (-2) * First place the three negatives.	

<table>
<tr><td>* Next perform the operation, take away 2 negatives. Thus –1 is the answer.</td><td></td></tr>
<tr><td colspan="2">Be sure to explain to the students that this is simply a way to model what is going on in a problem, and why taking away negatives is like adding. This is not a way to always perform subtraction. Scaffold into the idea of working with larger numbers, “Do we have more positives or negatives? So our answer will be ____, now subtract as usual, smaller from larger.”</td></tr>
</table>

Option 2: Using the Number Line

Materials: white boards, numbers line inserts

Description:

<table>
<tr><td colspan="2">Pass out the page protector white boards with number line inserts (shown on next page), to each student. Allow them to explore the number line, discuss the positive and negative directions.</td></tr>
<tr><td>Walk students through a few example problems, having them physically walk their fingers from one number to another.</td><td>Ex. –3 – (-4)
Start on the negative 3. Describe that taking away negatives is the same as making it more positive. So, walk 4 places in the positive direction. Land on the 1.

</td></tr>
<tr><td>This can also be done by drawing one arrow from 0 to negative 3. Then move four spots in the negative direction, from –3 to –7. Finally, count how long the arrows are together, the total distance traveled. 3 + 4 = 7</td><td>
</td></tr>
<tr><td>Give the students more examples.</td><td>-7 - 2   4 - (-2)   6 - (-5)   -5 - 8</td></tr>
<tr><td colspan="2">Scaffold into the idea of working with larger numbers, “Do we have more positives or negatives? So our answer will be ____, now subtract as usual, smaller from larger.”</td></tr>
</table>

~ Options: you can also pass out markers and have the students draw the arrows, or place + and – signs on the appropriate sides

<u>Option 3: Walking the Number Line</u>

Materials: walking number line (large green felt number line)

Description:

<table>
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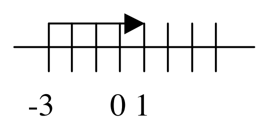
<tr><td>Walk students through a few example problems, having them physically walk from one number to another. If the number line is on the board, you can show the jumping of the numbers, (shown at right).</td><td>Ex. –3 – (-4)
Start on the negative 3. Describe that taking away negatives is the same as making it more positive. So, walk 4 places in the positive direction. Land on the 1.
</td></tr>
<tr><td colspan="2">Scaffold into the idea of working with larger numbers, “Do we have more positives or negatives? So our answer will be ____, now subtract as usual, smaller from larger.”</td></tr>
</table>

Lesson 2.6

Title:	The Distributive Property
Standards Addressed:	1.0, 4.0
Length of Activity:	15 min
Materials Used:	Algebra tiles, overhead tiles (for a shorter activity, do only on overhead)

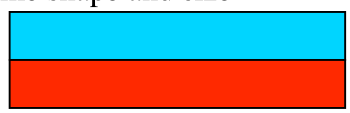
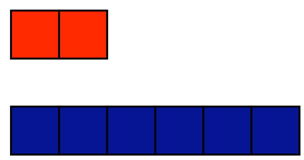
Description:

Pass out algebra tiles to each student or student pairs.	(see lesson 1.1 for description of tiles)
Present a model of the distributive property. * Lay of the model x + 1	Ex. 2(x + 1) 
* Explain that this model is 2 sets of (x + 1). This model creates a rectangle with length (x + 1) and width 2. The resulting area is 2x's and 2 1's, or 2x+2. Perform more examples (using x and y).	2 
Work into the idea of the process for distribution.	$2(x + 1)$ $\quad 2 * x + 2 * 1 = 2x+2$

Lesson 2.7

Title:	Combining Like Terms
Standards Addressed:	4.0
Length of Activity:	10 min
Materials Used:	overhead algebra models, (can use student models for a longer activity)

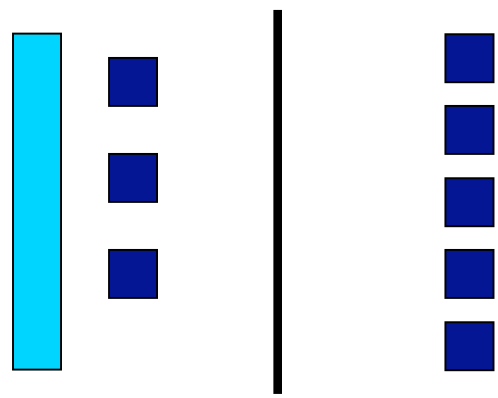
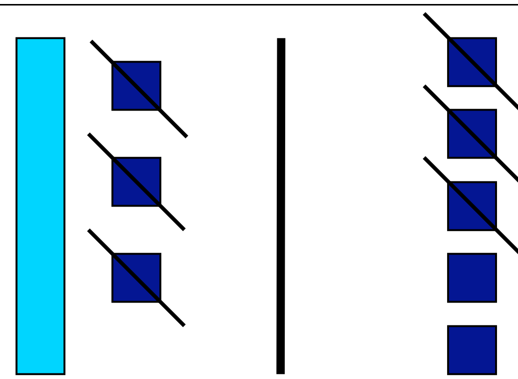
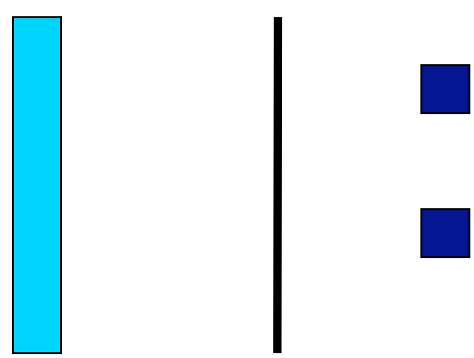
Description:

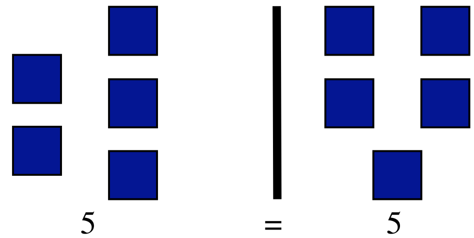
Lay out various algebra models on the overhead	(see lesson 1.1 for tile descriptions)
Explain what is a like term, having the same shape and size. Color (red versus other color) does not matter, it is just the sign of the term.	x and–x are like terms because they are the same shape and size 
 2y and y are like terms because they are the same shape and size	-2 and 6 are like terms because they are the same shape and size 

Lesson 3.1

Title:	Solving Equations Using Addition and Subtraction
Standards Addressed:	5.0
Length of Activity:	15 – 20 min
Materials Used:	Algebra models, overhead models

Description:

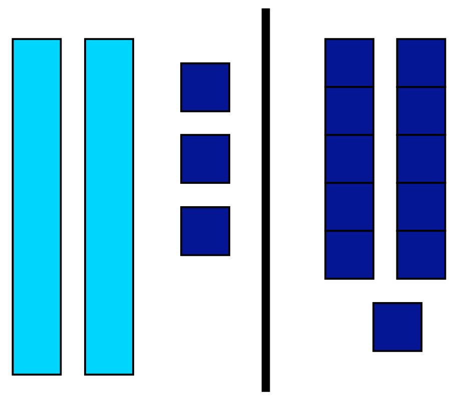
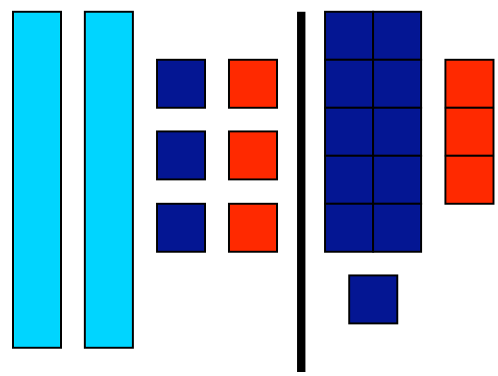
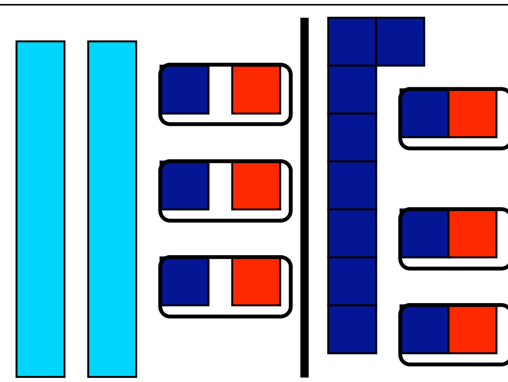
Pass out algebra models to each student or student pair	(see lesson 1.1 for tile descriptions)
Walk students through an example, (students can either separate two sides of the equation on their desk, or use a paper or white board with a vertical line down the center to represent the equal sign).	Ex. x + 3 = 5 
Subtract 3 from both sides	
Read what is left, x = 2	

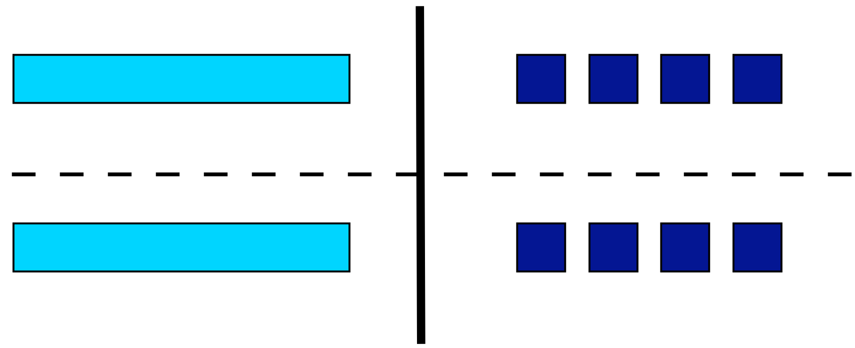
Plug back in x=2 to check	
Continue with further examples.	y – 1 = 2 -x + 5 = 7 x + 2 = 7

Lesson 3.3

Title:	Solving Multi-Step Equations
Standards Addressed:	5.0
Length of Activity:	15 - 20 min
Materials Used:	Algebra models, overhead models

Description:

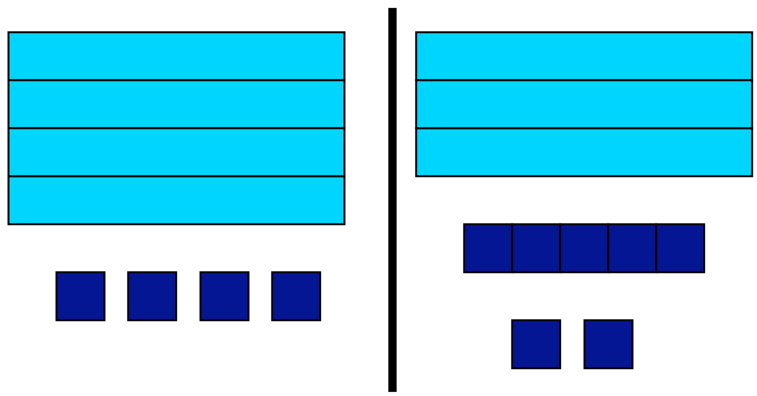
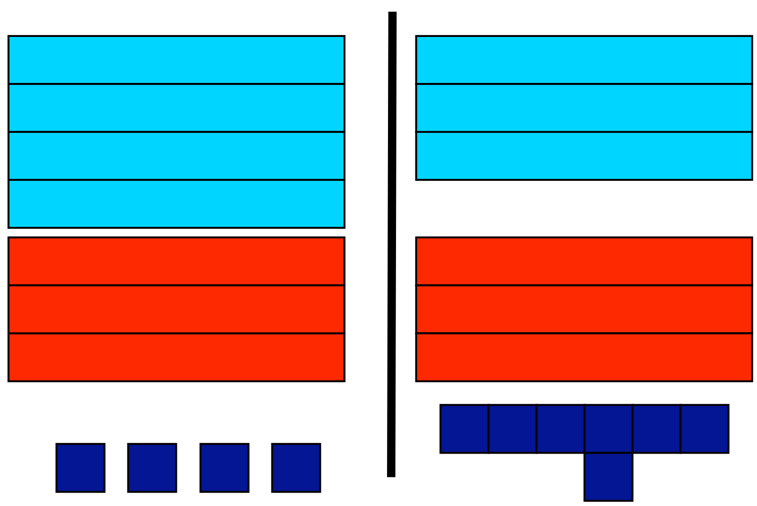
Pass out algebra models to each student or student pair	(see lesson 1.1 for tile descriptions)
Walk students through an example, (students can either separate two sides of the equation on their desk, or use a paper or white board with a vertical line down the center to represent the equal sign).	Ex. 2x + 3 = 11 
Subtract 3 from both sides of the equation (by adding –3 units to each side).	
Remove the zero pairs, (remind students that –1 + 1 = 0).	

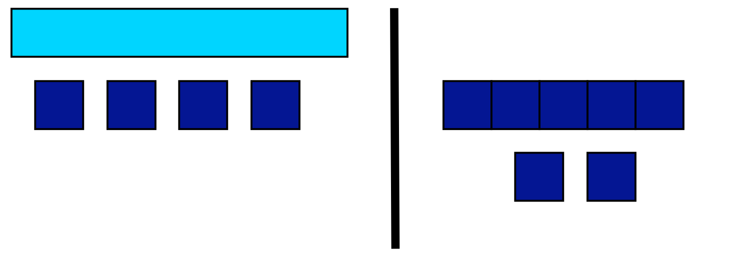
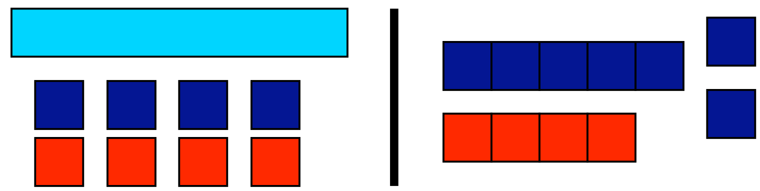
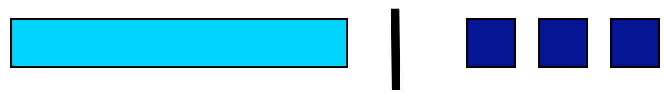
<table>
<tr><td>Divide both sides by 2, split evenly each side into two piles</td><td></td></tr>
<tr><td colspan="2">Therefore each x = 4. This can be plugged back in to check. Continue with more examples.</td></tr>
</table>

Lesson 3.4

Title:	Solving Equations with Variables on Both Sides
Standards Addressed:	5.0
Length of Activity:	15 – 20 min
Materials Used:	Algebra models, overhead models

Description:

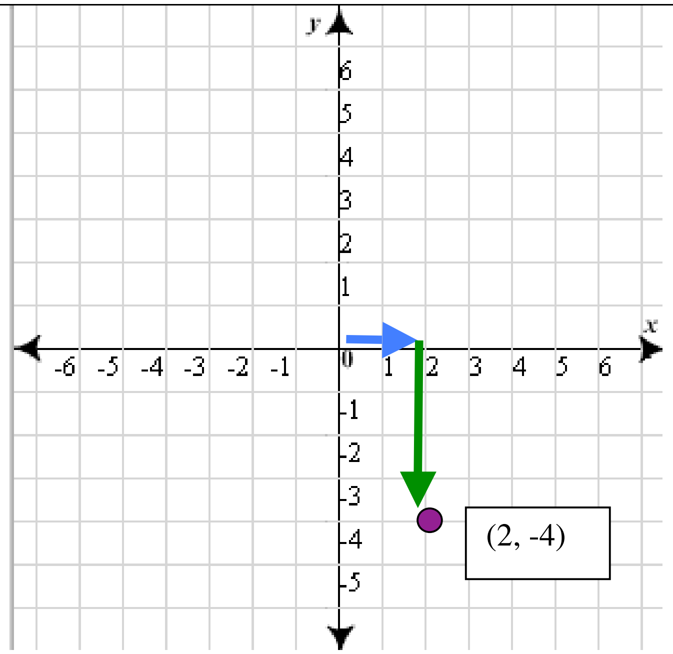
Pass out algebra models to each student or student pair	(see lesson 1.1 for tile descriptions)
Walk students through an example, (students can either separate two sides of the equation on their desk, or use a paper or white board with a vertical line down the center to represent the equal sign).	Ex. $4x + 4 = 3x + 7$ 
Get all of the x tiles on the same side. Subtract 3x from both sides.	

After removing zero pairs, we are left with 1x + 4 = 7	
To isolate the variable, subtract 4 from both sides.	
After removing zero pairs, we can see x = 3.	
This can be plugged back in to check. Continue with more examples.	

Lesson 4.1

Title:	The Coordinate Plane
Standards Addressed:	none
Length of Activity:	10 min
Materials Used:	White boards with coordinate plane inserts, dry erase pens, eraser rags

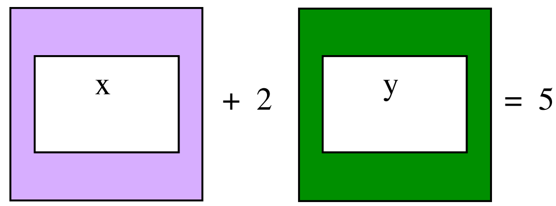
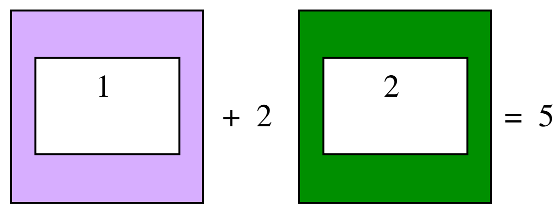
Description:

<table>
<tr><td colspan="2">Briefly introduce the coordinate plane and key terms after passing out the white boards with coordinate plane inserts.</td></tr>
<tr><td>Given a point, have the students start at the origin, and walk with their fingers the direction of the x-coordinate and then the direction of the y-coordinate. Once at the point, they can label it.
Ex. (2, -4)</td><td>
</td></tr>
</table>

Lesson 4.2

Title:	Graphing Linear Equations
Standards Addressed:	6.0, 7.0
Length of Activity:	10 min
Materials Used:	Felt pockets with index cards

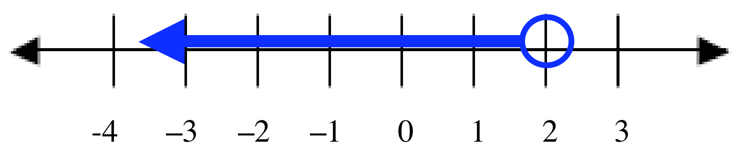
Description:

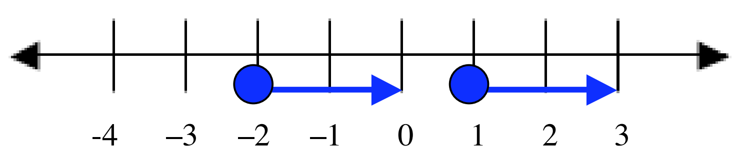
Tape or clip the felt pockets to the board. Write the equation on the board as is, with correct variable cards in the pockets. The pockets are to emphasize that a variable represents a value, and what substitution means.	
Question: is (1, 2) a solution of x + 2y = 5. Replace the x card with a 1 card, and replace the y card with a 2 card.	
Solve, using order of operations	1 + 2(2) = 5 1+ 4 = 5 5 = 5 true, so is a solution

Lesson 6.1

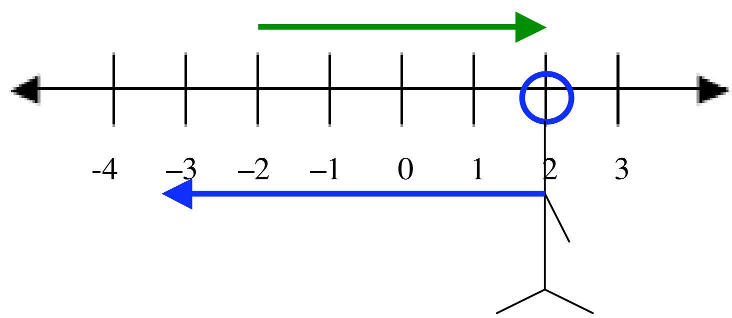
Title:	Solving Inequalities Using Addition and Subtraction
Standards Addressed:	5.0
Length of Activity:	20 min
Materials Used:	Large walking number line

Description:

<table>
<tr><td colspan="2">Mount the number line by clipping it or taping to the top of your board. Just write the numbers on the line. Review the positive and negative directions and <, > signs.</td></tr>
<tr><td colspan="2">Review the positive and negative directions and <, > signs.
Have the graph on the wall, and demonstrate the directions as shown below. Students can come to participate as well.</td></tr>
<tr><td>• Given x<2, ask the students to give the verbal description.
• Stand at the 2 spot on the number line
• Ask the students which direction you should walk, toward the positive or toward the negative.
Discuss, what does the less than mean? Is 2 the greatest number or the smallest?</td><td>x < 2   All real numbers less than 2


~ Continue with further examples of each sign, <, >, ≤, ≥
~ Have the students come up to perform examples as well</td></tr>
<tr><td>Demonstrating the Addition Property of Inequality:
For all real numbers a, b, and c
If a<b, then a+c<b+c
If a>b, then a+c>b+c</td><td>

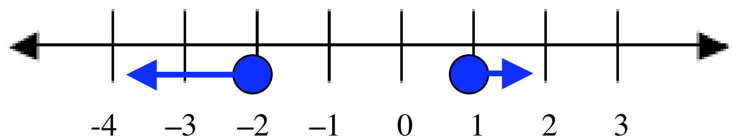
For a < b,
• Have two students stand at two different places on the number line, ex. –2 and 1, so student 1 is at a</td></tr>
</table>

<table>
<tr><td></td><td>lower position than student 2
• Let c = 2, so have both students move two spots in the positive direction
• Ask the students, is student 1 still at a lower position than student 2?
• This shows that if you add the same amount, a will still be lower than b.</td></tr>
<tr><td colspan="2">The process is the same for a > b and with the Subtraction Property of Inequality. Have different students come up to demonstrate the properties.</td></tr>
<tr><td>For single step inequalities, such as -2 > n – 4
• Have a student stand at –2
• Discuss that we want to know n, not n-4, so we add four to both sides to get n by itself
• Have the student walk 4 spaces in the positive direction
• With the help of the class, have them place their arm in the direction the arrow should go</td><td>
</td></tr>
</table>

Lesson 6.2

Title:	Solving Inequalities Using Multiplication or Division
Standards Addressed:	5.0
Length of Activity:	15 – 20 min
Materials Used:	Large walking number line

Description:

<table>
<tr><td colspan="2">Mount the number line by clipping it or taping to the top of your board. Just write the numbers on the line. Review the positive and negative directions and <, > signs.</td></tr>
<tr><td>Demonstrating the Multiplication Property of Inequality:
For all real numbers a, b, and for $c > 0$
If $a<b$, then $ac<bc$
If $a>b$, then $ac>bc$</td><td>

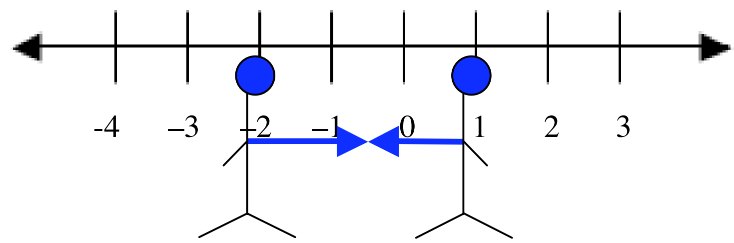
For $a < b$,

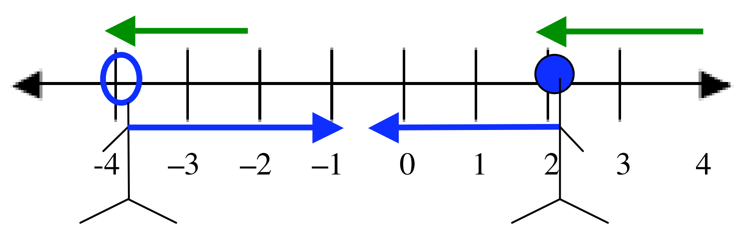
Have two students stand at two different places on the number line, ex. –2 and 1, so student 1 is at a lower position than student 2
Let $c = 2$, so have both students move two times their number
Ask the students, is student 1 still at a lower position than student 2?
This shows that if you multiply by the same amount, a will still be lower than b.
</td></tr>
<tr><td colspan="2">The process is the same for $a > b$ and with the Division Property of Inequality. Have different students come up to demonstrate the properties.</td></tr>
</table>

Lesson 6.4

Title:	Solving Compound Inequalities Involving “And”
Standards Addressed:	5.0
Length of Activity:	10 min
Materials Used:	Large walking number line

Description:

<table>
<tr><td colspan="2">Mount the number line by clipping it or taping to the top of your board. Just write the numbers on the line. Review the positive and negative directions and <, > signs.</td></tr>
<tr><td>Introduce the term compound inequality.
Given the example $x \geq -2$ and $x \leq 1$
• Have one student come up and demonstrate the inequality $x \geq -2$, using their arms as the arrows
• Have another students come up and demonstrate the inequality $x \leq 1$ using their arms as the arrows
• Explain that the space between –2 and 1 is the answer to the problem</td><td>

Have the students link arms to show a solid line between –2 and 1</td></tr>
<tr><td>For single step compound inequalities, such as $-2 < x + 2 \leq 4$
• Have a student stand at –2 and another at 4
• Discuss that we want to know x, not x+2, so we subtract 2 from all sides of the inequality to get x by itself
• Have both students walk 2 spaces in the negative direction
• With the help of the class, have them place their arm in the direction the arrow should go, then link arms to produce an answer</td><td>

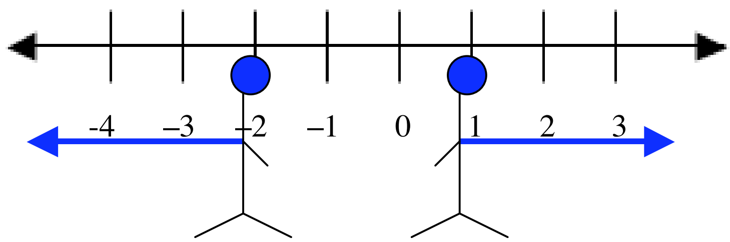
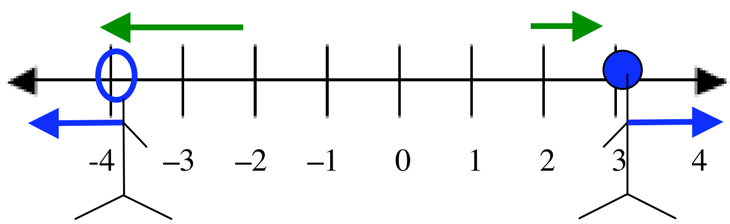
The solution shown is $-4 < x \leq 2$

Discuss that this answer includes 2, but not -4</td></tr>
</table>

Lesson 6.5

Title:	Solving Compound Inequalities Involving "Or"
Standards Addressed:	5.0
Length of Activity:	10 min
Materials Used:	Large walking number line

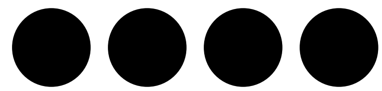
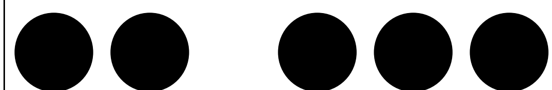
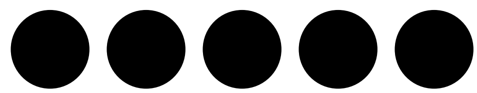
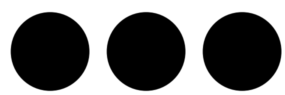
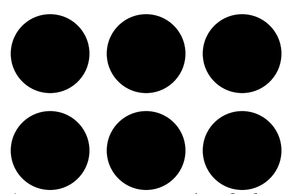
Description:

<table>
<tr><td colspan="2">Mount the number line by clipping it or taping to the top of your board. Just write the numbers on the line. Review the positive and negative directions and <, > signs.</td></tr>
<tr><td>Given the example $x \le -2$ and $x \ge 1$
• Have one student come up and demonstrate the inequality $x \le -2$, using their arms as the arrows
• Have another students come up and demonstrate the inequality $x \ge 1$ using their arms as the arrows</td><td>
Discuss that both arrows are the solution, x could be in either section.</td></tr>
<tr><td>For single step compound inequalities, such as $-2 > x + 2$ or $x - 1 \ge 2$
• Have a student stand at –2 and another at 2
• Discuss that we want to know x, not x+2 or x-1, so we solve each inequality separately to get x by itself
• Have half the class help each student to solve their inequalities for x</td><td>
Discuss that both arrows are the solution, x could be in either section.
Discuss that this answer includes 3, but not -4</td></tr>
</table>

Lesson 8.1

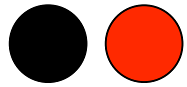
Title:	Multiplication Properties of Exponents
Standards Addressed:	2.0
Length of Activity:	15 – 20 min (or longer if you wish)
Materials Used:	black/red counters, overhead counters (if desired)

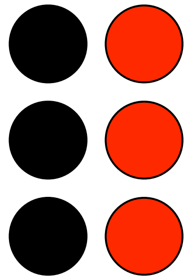
Description:

Pass out the counters to each student or student pair	
Review examples of powers represented with the counters. (note: color doesn't matter)	Ex: 3^4  = 4, 3's or 3*3*3*3
Demonstrate an example of the Product of Powers Property. Ex. a^2 * a^3 After a few example, scaffold into the formula: a^m * a^n = a^(m+n)	 Lay out a^2 Lay out a^3  Ask how many a's are being multiplied all together = 5 a's or a^5 (a*a*a*a*a)
Demonstrate an example of the Power of a Power Property. Ex. (a^3)^2 After a few example, scaffold into the formula: (a^m)^n = a^(m*n)	 Lay out a^3 Since it is squared, lay out 2 sets of a^3  There are a total of 6, a's = a^6

<table>
<tr>
<td>Demonstrate an example of the Power of a Product Property.
Ex. (a * b)^3

* Note: This is the only property where color matters.

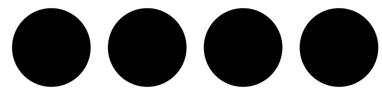
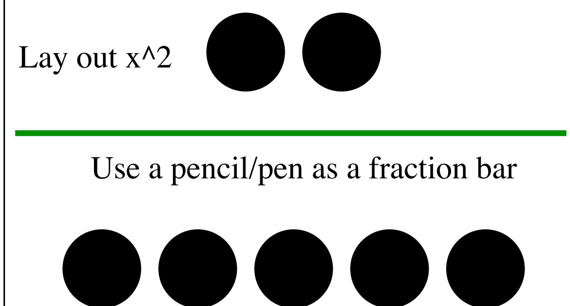
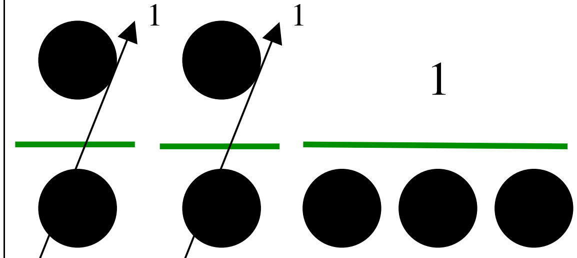
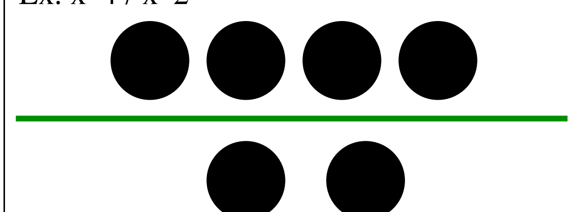
After a few example, scaffold into the formula:
(a * b)^m = a^m * b^m</td>
<td>Let black = a and red = b

Lay out (a * b)

Since the quantity is cubed, lay out 3 sets

Group by type:
= a^3 * b^3</td>
</tr>
<tr>
<td colspan="2">As you go, do as many examples of each property with the students as you wish. Students may want to come to the overhead to demonstrate.</td>
</tr>
</table>

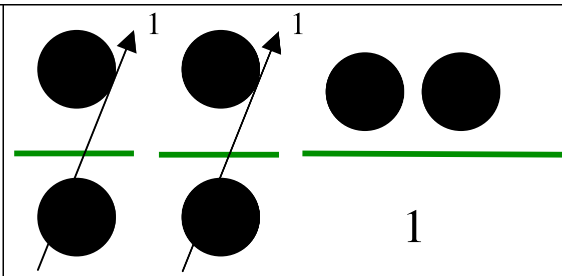
Lesson 8.4

Title:	Division Properties of Exponents
Standards Addressed:	2.0
Length of Activity:	10 – 15 min
Materials Used:	black/red counters, overhead counters (if desired)

Description:

Pass out the counters to each student or student pair	
Review examples of powers represented with the counters (if necessary) (note: color doesn't matter)	Ex: 3^4  = 4, 3's or 3*3*3*3
Demonstrate an example of the Quotient of Powers Property. Ex. $\frac{x^2}{x^5}$ Have students explore how they could reduce this fraction, giving them a few minutes to work.	Lay out x^2  Use a pencil/pen as a fraction bar Lay out x^5
After some time, remind students that anything over itself = 1. Show the separating of the fraction into x/x pairs. ~Be sure to note to the students that there is not a zero left in the numerator, there is a 1.	 1 * 1 * 1/x^3 = 1/x^3
Do another example where the exponent on the numerator is larger than that of the denominator.	Ex. x^4 / x^2 

<table>
<tr>
<td>As before, allow the students to reduce the fraction into x/x pairs.

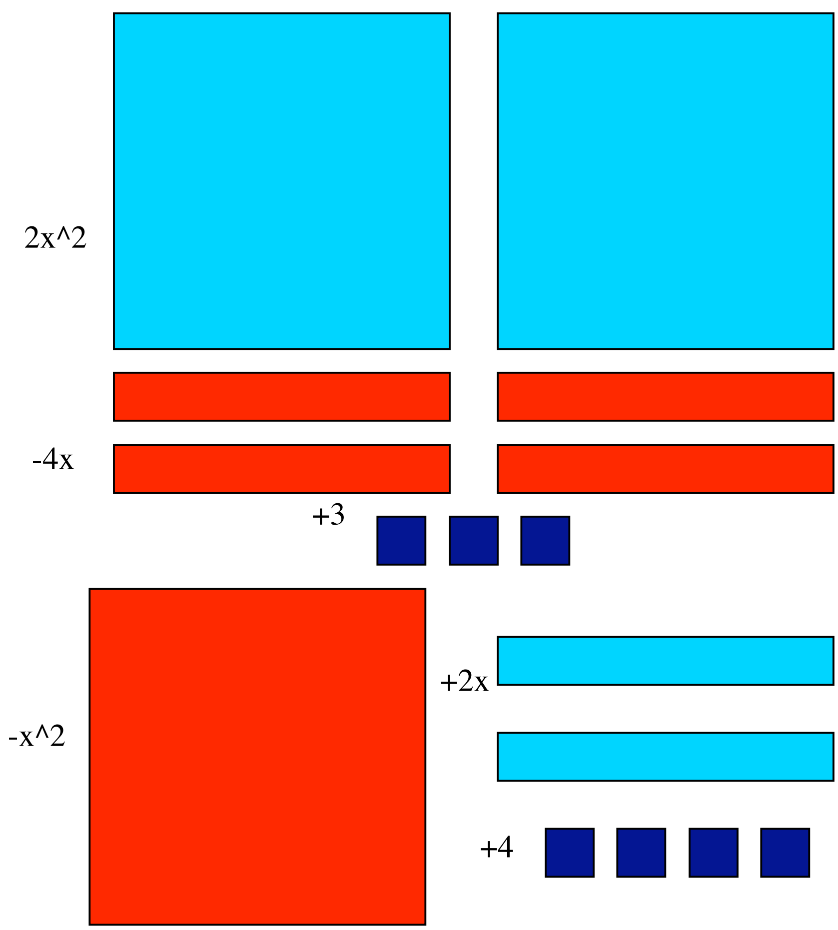
~Be sure to note to the students that there is not a zero left in the denominator, there is a 1.</td>
<td>
1 * 1 * x^2/1 = x^2</td>
</tr>
<tr>
<td colspan="2">Scaffold into formula: x^m / x^n = x^(m-n)</td>
</tr>
</table>

Lesson 10.1

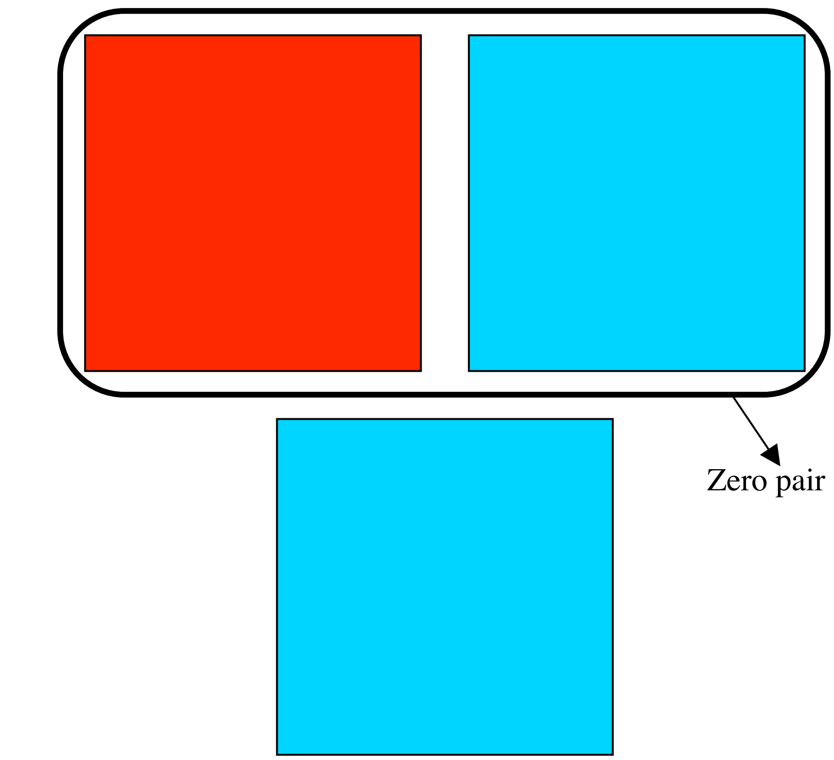
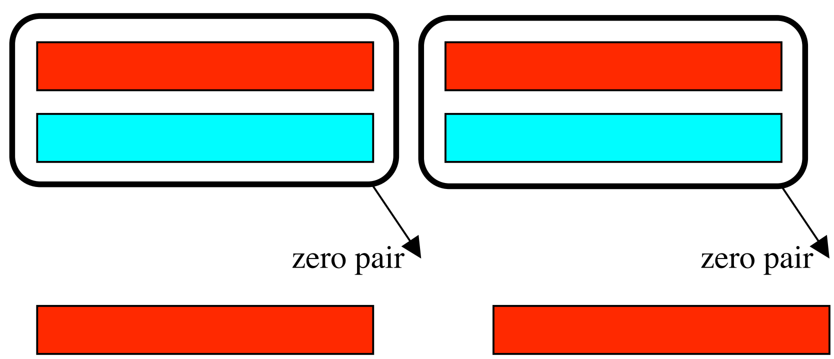
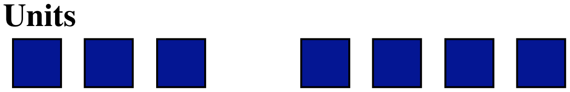
Title:	Adding and Subtracting Polynomials
Standards Addressed:	10.0
Length of Activity:	20 min (or longer if desired)
Materials Used:	Algebra models, overhead models (if desired)

Description:

<table>
<tr><td colspan="2">After introducing the degree of a polynomial and how to label it my degree and number of terms, pass out the algebra models to each student or student pairs.</td></tr>
<tr><td>Use the algebra models to introduce the idea of adding polynomials.

Ex. (2x^2 – 4x + 3) + (-x^2 + 2x + 4)</td><td>Lay out each polynomial, careful to note negatives

</td></tr>
</table>

<table>
<tr><td>Combine like terms, looking at each term separately and simplifying any zero pairs.

1x^2 left</td><td>x^2's

</td></tr>
<tr><td>-2x left</td><td>x's

</td></tr>
<tr><td>7 units left</td><td>Units
</td></tr>
<tr><td>Read what is left</td><td>x^2 –2x + 7</td></tr>
<tr><td colspan="2">Continue with more examples as desired. Emphasize the idea of what is a like term, using the example of the models as a visual representation, (like terms are the same shape and size).</td></tr>
<tr><td colspan="2">Also perform an example of subrtacitng polynomials. The process is the same, but careful to remind the students to distribute the negative. This can be done by laying out the second polynomial, then flipping over all tiles to the opposite side.</td></tr>
</table>

Lesson 10.2

Title:	Multiplying Polynomials
Standards Addressed:	10.0
Length of Activity:	15-20 min, or longer (if desired)
Materials Used:	Algebra models, overhead algebra models (if desired)

Description:

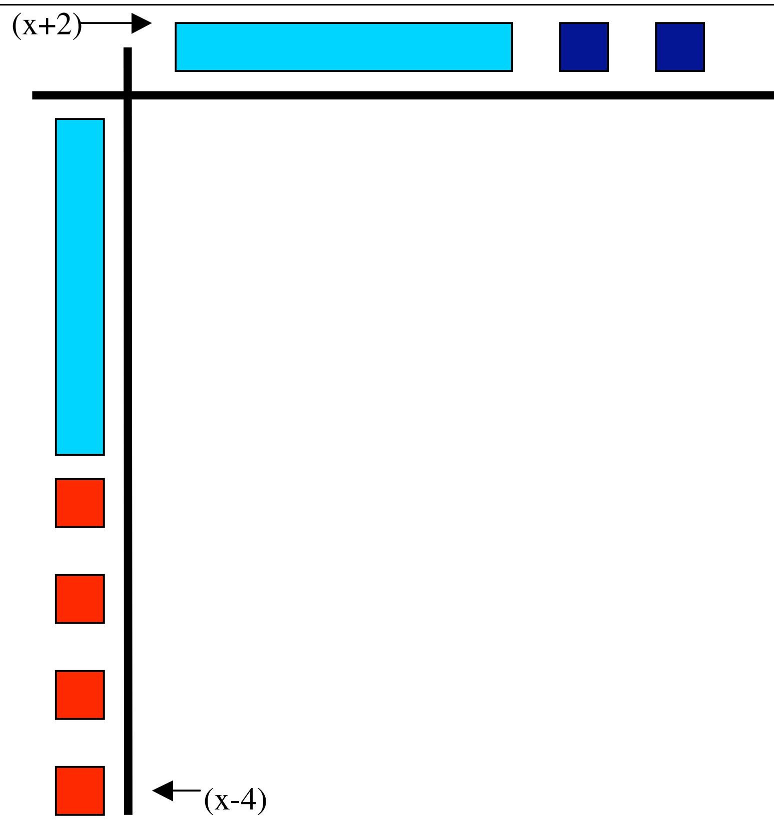
Pass out the algebra models to each students or student pair

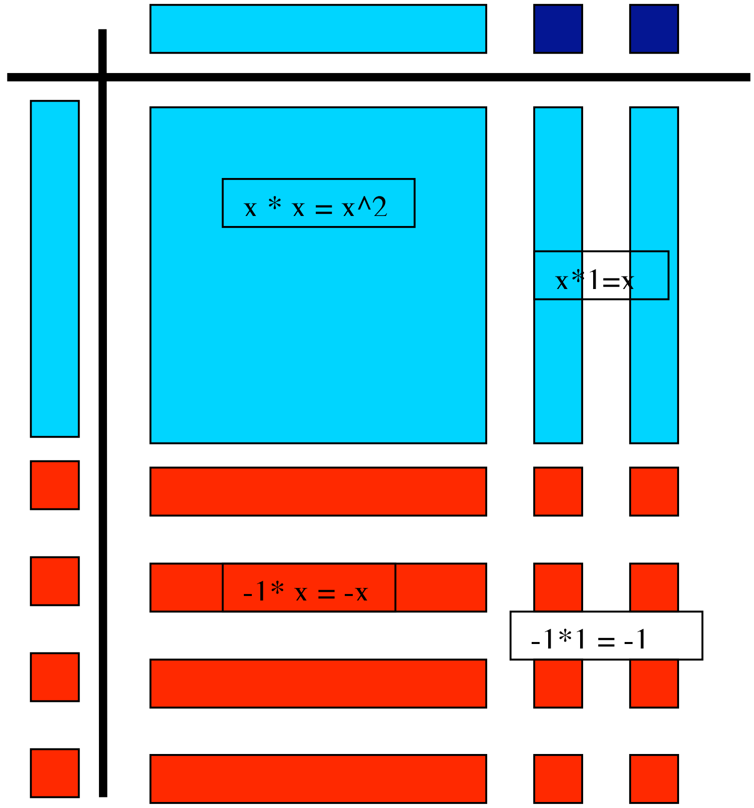
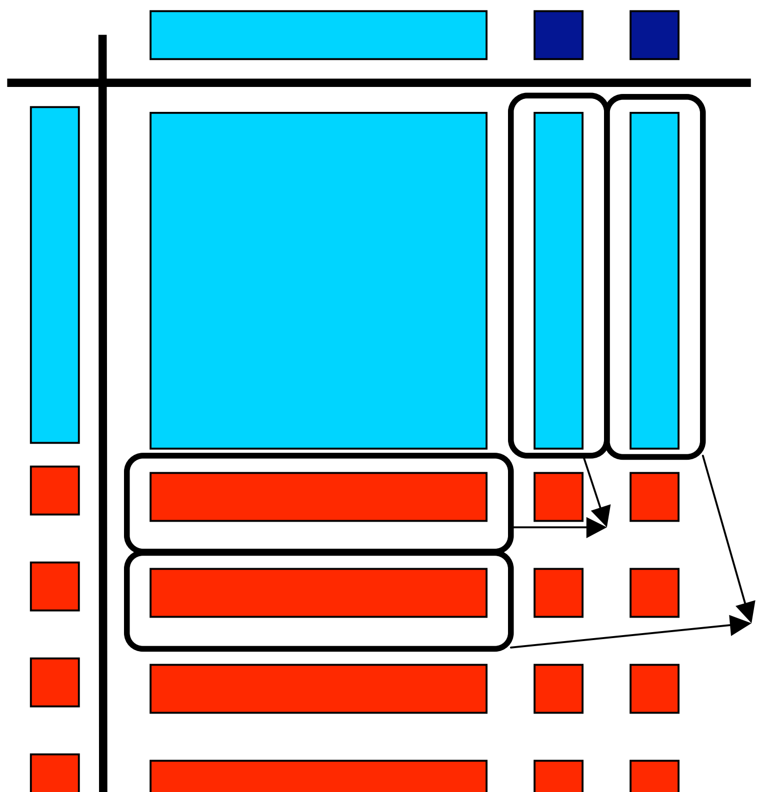
Multiplying polynomials can be expressed with algebra models through an area model. It is similar to multiplying the area of a rectangle.

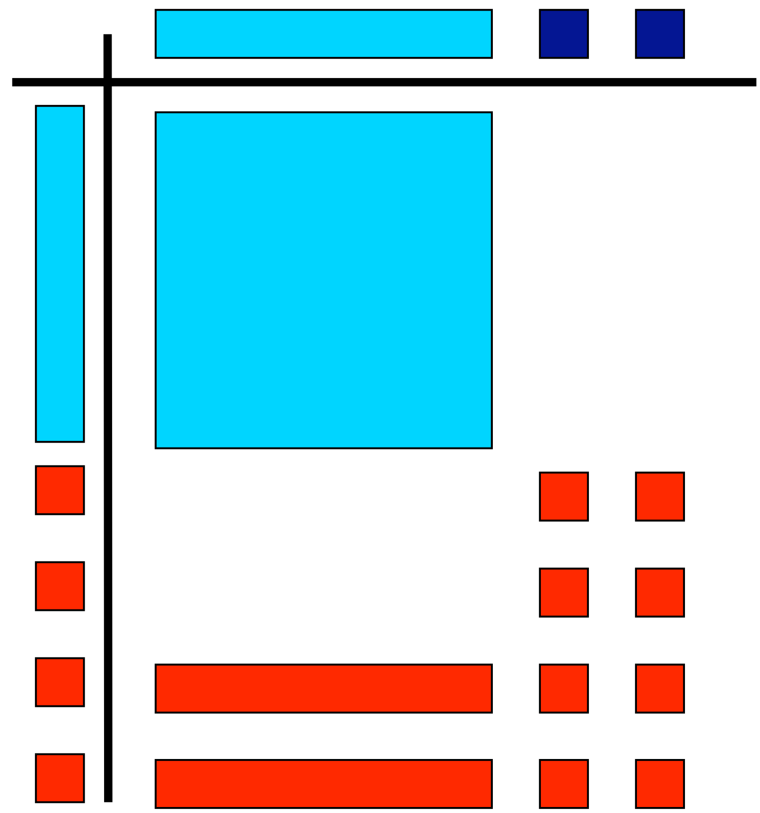
Ex. (x+2)(x-4)

* Note: have the students draw the shown grid on their paper, or make a copy of the grid on page 123 of the accompanying Algebra Models workbook and insert them in the whiteboards.

Lay out each of the polynomials on the tops of the grids.



Multiply each factor of the polynomial on the left with each on the right.	
Cancel out any zero pairs.	
Read what remains.	

<table>
<tr><td>x^2 – 2x - 8</td><td></td></tr>
<tr><td colspan="2">Continue with more examples. Also, work up to a problem such as (x+y)(x+y) and use the green xy tile. Begin to assign labels to each of the peices as you scoffold into the FOIL or other meathods for multiplying.</td></tr>
</table>

Ideas for expansion:

If you desire the activity to be longer, you may also have the students draw out the model they built and write out their answers to turn in. If FOIL or another method is to be taught on the same day, you can relate the idea of the area of each piece to each step in the FOIL process.

Lesson 10.3

Title:	Special Products of Polynomials
Standards Addressed:	10.0
Length of Activity:	15-20 min
Materials Used:	Algebra models, overhead algebra models (if desired)

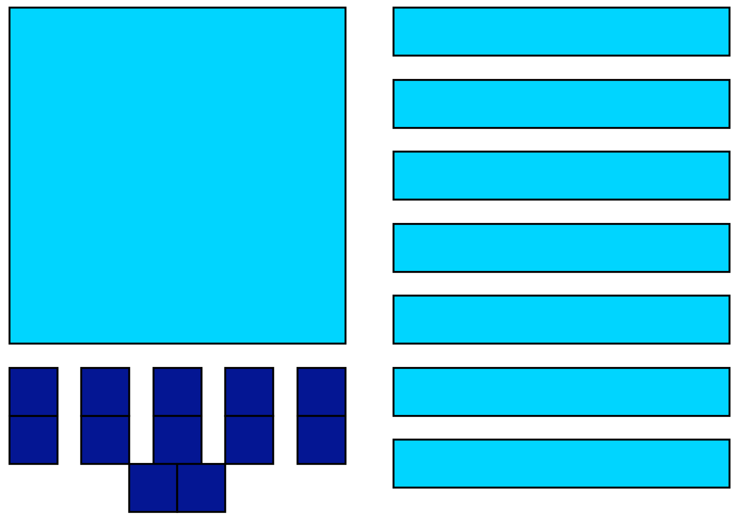
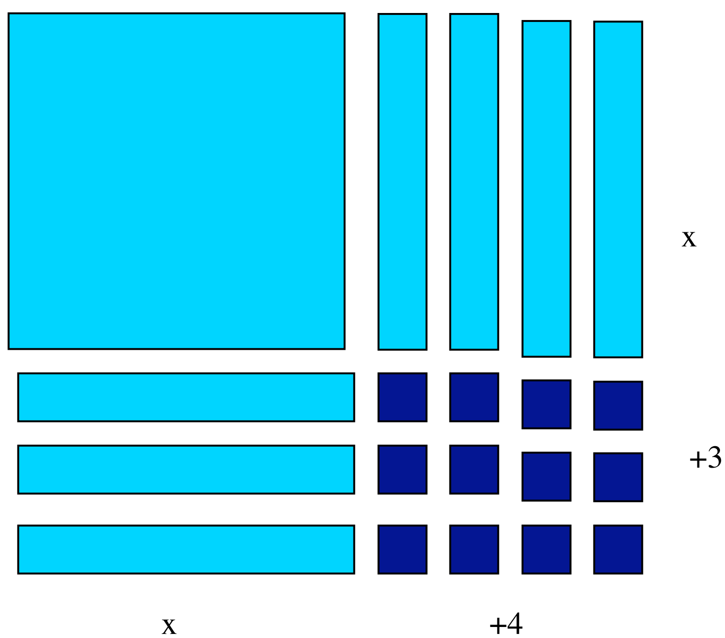
Description:

- Please refer to lesson 10.2 to learn how to multiply polynomials. Repeat this same process with an example of each of the special product patterns to demonstrate how and why they are a pattern.
- Special Product Patterns:
    - $(a+b)(a-b) = a^2 – b^2$
    - $(a+b)^2 = a^2 + 2ab + b^2$
    - $(a – b)^2 = a^2 – 2ab + b^2$

Lesson 10.5

Title:	Factoring x^2 + bx+ c
Standards Addressed:	11.0, 14.0
Length of Activity:	15 – 20 min
Materials Used:	Algebra models, overhead algebra models (if desired)

Description:

Pass out a set of algebra models to each student or student pair.	
Lay out a trinomial in algebra models. Ex. x^2 + 7x + 12	
Make a rectangle out of the tiles, make sure that the dimensions both lengths and both widths match. Then, read the dimensions of the length and width of the rectangle. * Note: This is the opposite process of lesson 10.2	
Find the factors by reading the length and width: (x+3)(x+4)	

Lesson 10.6

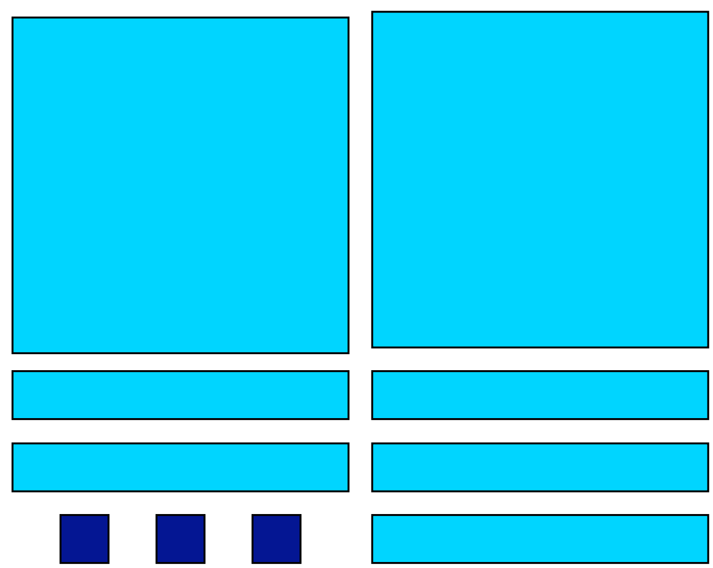
Title:	Factoring ax^2 + bx+ c
Standards Addressed:	11.0, 23.0
Length of Activity:	15 – 20 min
Materials Used:	Algebra models, overhead algebra models (if desired)

Description:

Pass out a set of algebra models to each student or student pair.

Lay out a trinomial in algebra models.

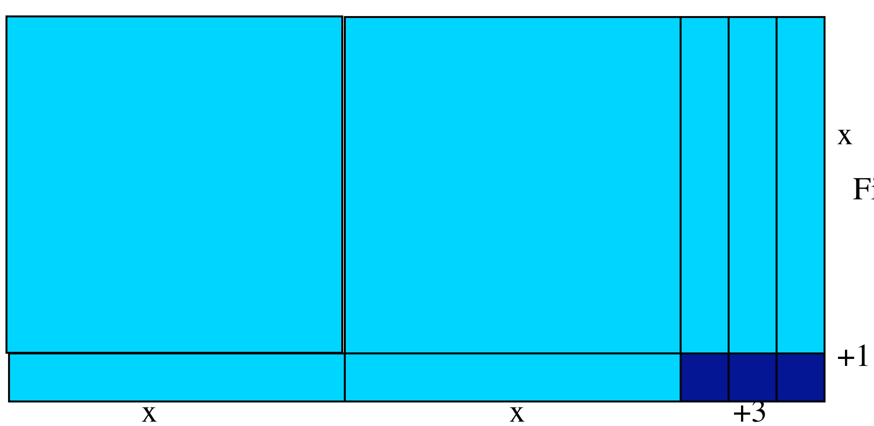
Ex. 2x^2 + 5x + 3



Make a rectangle out of the tiles, make sure that the dimensions both lengths and both widths match.

Then, read the dimensions of the length and width of the rectangle.

* Note: This is the opposite process of lesson 10.2



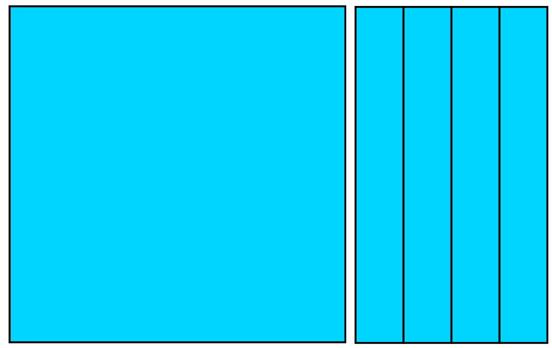
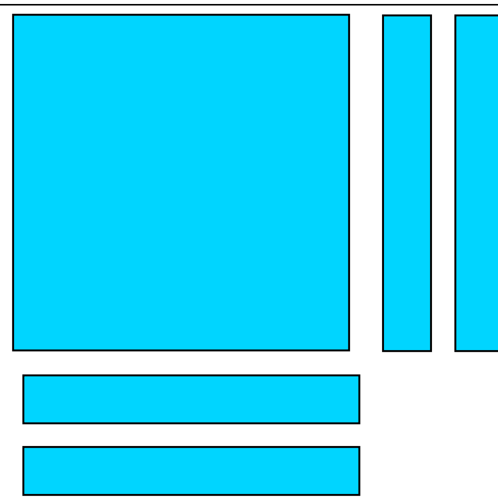
Find the factors by reading the length and width:

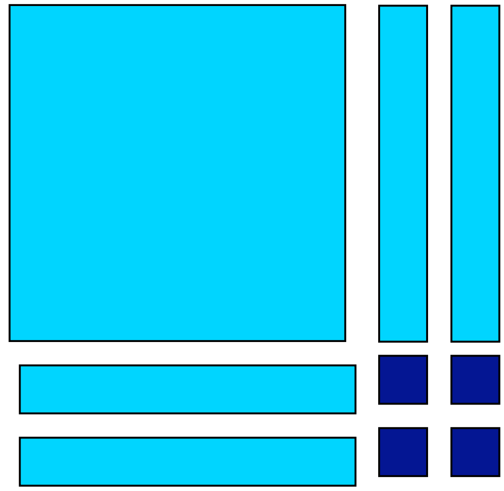
(2x+3)(x+1)

Lesson 12.5

Title:	Completing the Square
Standards Addressed:	14.0, 19.0
Length of Activity:	10 min
Materials Used:	Algebra models, overhead algebra models (if desired)

Description:

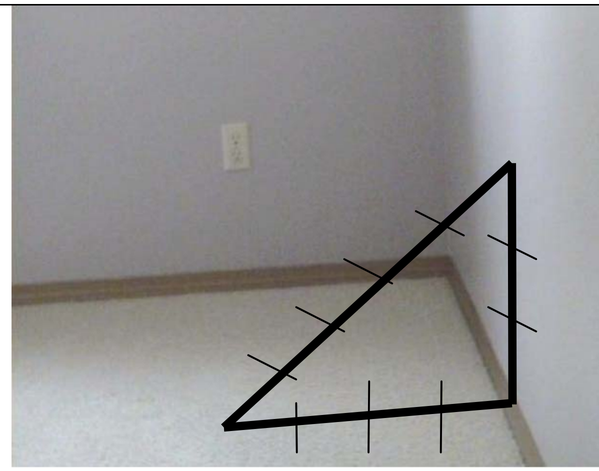
To demonstrate what a square trinomial means, lay out an x^2 tile and an even number of x tiles. Ex. $x^2 + 4x$	
Move the tiles to form part of a square, with an even number of x's on both sides of the x^2	

Fill in the remaining part of the square with units to create a perfect square. Read the dimensions of the length and width, which will always be equal since it is a square. The units will always be a square, and the number of x’s on each side the square root of that number.	 $= (x + 2)^2$

Lesson 12.6

Title:	The Pythagorean Theorem and Its Converse
Standards Addressed:	24.2
Length of Activity:	10 min
Materials Used:	Large walking number line

Description:

To introduce the Pythagorean Theorem, have students use the large number line to explore Pythagorean triples. Have 2 students come to the front of the room and place the number line against the wall, to create the 90-degree angle. Have them see if they can create what they think is a right triangle, where each side is a whole number.	
Once they have measurements, have them plug them into the formula to check. Have several students try.	
Next, have students try common Pythagorean triples so they can see how exact they work.	Ex. 3, 4, 5 5, 12, 13

Lesson 12.7

Title:	The Distance Formula
Standards Addressed:	2.0, 25.0
Length of Activity:	10 min
Materials Used:	Large walking number line

Description:

To introduce the distance formula, relate it to lesson 12.6, Pythagorean Theorem. Use the same process of the number line as a right triangle against the wall, substituting the “c” or hypotenuse for “d,” distance.